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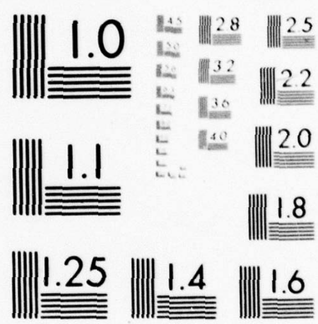
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This report describes research on redundancy management in digital flight control systems. The emphasis is on the properties, techniques, and requirements associated with the operations of monitoring and voting and their effects on the closed loop system operation when asynchronous sampling is used. These topics are among those being studied in the real-time simulation facilities of the Air Force Flight Dynamics Laboratory's Digital Avionics Information System (DAIS).

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20. Abstract

Part I is concerned primarily with the monitoring operation for quad-redundant input signals, and utilizes assumptions compatible with the DIAS program. The redundant signals are assumed to change one-at-a-time and the monitor uses cross-channel comparisons and a binary test for deciding whether two signals are within tolerance of each other as the basis for its fault-detection algorithm. All possible relationships among quad- or tri-redundant signals subjected to such comparisons are tabulated and grouped into "patterns". Patterns are in turn used to deduce "Keys", which are useful for understanding the way the signal relationships change with time. A basic algorithm for monitoring based on the above characterization is described, tested, and compared briefly with other currently used algorithms.

Part II presents three extensions to a previously reported model for closed loop flight control systems that have dual-redundant, asynchronous digital controllers. The original model had the same sample rate for each controller and a fixed time skew, or offset between their respective sample times. The first extension, the Multirate Model, allows for separate sampling rates for the external inputs (pilot input, wind gusts, etc.) and the digital controllers. The next extension, the Delay Model, allows for computational delays due to the time required for data conversions and control-output computations. The third extension, the Output-Averaging Model, provides for averaging the control outputs produced by each of the redundant controllers, rather than always selecting the output of the same controller, which is the scheme followed in all the other models. Equations and, in some cases, FORTRAN programs are described for both deterministic and statistical analyses of the inherent errors as calculated with the models.

TABLE OF CONTENTS

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TABLE OF CONTENTS (CONT.)

TABLE OF CONTENTS (CONT.)

LIST OF ILLUSTRATIONS

PART I

LIST OF ILLUSTRATIONS (CONT.)

LIST OF ILLUSTRATIONS (CONT.)

LIST OF TABLES

ABSTRACT

This report describes research on redundancy management in digital flight control systems. The emphasis is on the properties, techniques, and requirements associated with the operations of monitoring and voting and their effects on the closed loop system operation when asynchronous sampling is used. These topics are among those being studied in the real-time simulation facilities of the Air Force Flight Dynamics Laboratory's Digital Avionics Information System (DAIS).

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Part I

Characterization and Monitoring of Redundant Signals

1.0 INTRODUCTION AND SUMMARY

The inputs to redundant digital flight control systems may themselves be redundant.[1] In such cases two operations may be performed on each set of these redundant input signals. First, a monitoring algorithm separates the faulted from the unfaulted signals. Second, a voting algorithm produces a single best estimate of the true value of the signal, based on the signals that the monitoring algorithm declares as unfaulted. This report (Part I) is concerned primarily with the monitoring operation.

The assumptions made herein are compatible with (but not necessarily identical to or limited to) those that constrain the real-time simulation facilities of the Air Force Flight Dynamics Laboratory's Digital Avionics Information System (DAIS) advanced development program. The DAIS program was established jointly by two Air Force Laboratories as an approach for reducing the spiraling life-cycle costs of avionics. The overall objectives of the program are to develop and evaluate a set of standardized hardware and software modules (core elements) that can be configured for use in a wide variety of aircraft types and missions. The modules include computers, multiplex data bus hardware, controls and displays hardware, and software.

The DAIS hardware modules will be part of a Flight Engineering Facility, which will be used for real-time simulation and evaluation of digital flight control system performance, interactions between flight control and avionics,

and the pilot interface with the system. The basic DAIS flight control system is quad redundant. Each of the digital flight control processors is programmed in an identical manner and operates on redundant, sampled input data, but the sampling is asynchronous, which means that the common, preprogrammed sampling rates may vary among the identical processors by small percentages. As a result, there are time skews in the input data sampling--approaching a full sample period in the worst case--and the processors' computations are close to each other but not bit-for-bit identical.[4]

This report concentrates on the monitoring operation for quad-redundant input signals. The redundant signals are assumed to change one-at-a-time and the monitor uses cross-channel comparisons as the basis for its fault-detection algorithm.

Section 2 below describes a means for characterizing all possible relationships among quad- or tri-redundant signals subjected to a cross-channel monitoring scheme that uses a binary test for deciding whether two signals are within tolerance of each other. States are defined and tabulated. Then patterns are deduced and grouped into Keys, which are useful for understanding the way the state changes with time.

Section 3 describes a basic algorithm for the monitoring operation. The algorithm is based on the signal-pattern characterization of the previous section. Detailed flowcharts and explanations are given for the algorithm so that it may be easily implemented in software. A simpler algorithm is also mentioned and related to the basic algorithm.

Section 4 presents an evaluation of the basic algorithm, based on both computer simulation studies and comparisons with other currently used algorithms.

Section 5 contains the references.

2.0 SIGNAL CHARACTERIZATION

2.1 Assumed Sampling and Monitoring System

A typical digital flight control system may be represented as in Figure 1. When redundancy is employed, any or all of the components depicted may be replicated or multiplexed. This report assumes quad-redundant sensors and that the flight-control processor implements, in software, a monitoring algorithm to isolate the "good" (unfaulted) sensor inputs from the "bad" (faulted) ones.

The monitoring algorithm to be developed employs input-input comparison monitoring and additional logic operations. (See Figure 2.) For comparison monitoring, two signals, say x_i and x_j, are compared according to $|x_i - x_j| \leq$ Tolerance

The comparator output (one of the f_{ij} produced by the C's in Figure 2) for x_i and x_j has a value of 0 if the inequality is satisfied and 1 otherwise. The monitor logic implements an algorithm that uses both the signal values and the comparator outputs to eliminate those signals to be faulted; in general, the algorithm also requires storage of past history.

2.2 States and Patterns

Figure 3 shows a possible relationship among the redundant input signals x_1, x_2, x_3, and x_4. The bars in the figure indicate the allowable tolerance between signals. (Generally, this tolerance is fixed at a value determined by analysis of the system dynamics and sampling rates.) In this instance x_1, x_2, and x_3 are within tolerance of each other, as are x_1, x_2, and x_4; x_3 and x_4 are out-of-tolerance with

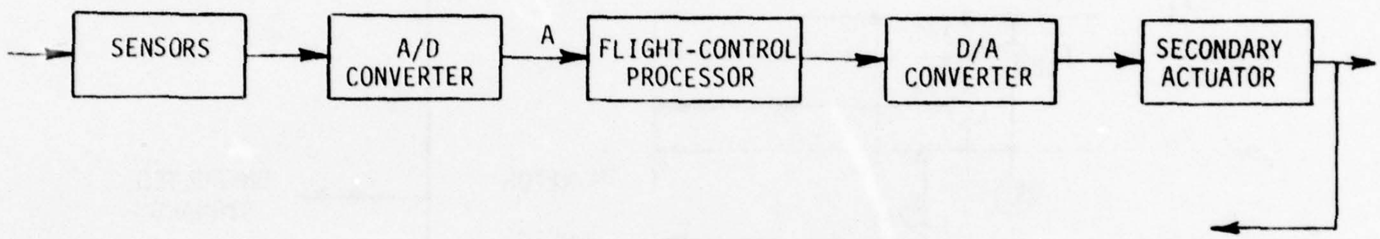


FIGURE 1. TYPICAL DIGITAL FLIGHT CONTROL SYSTEM

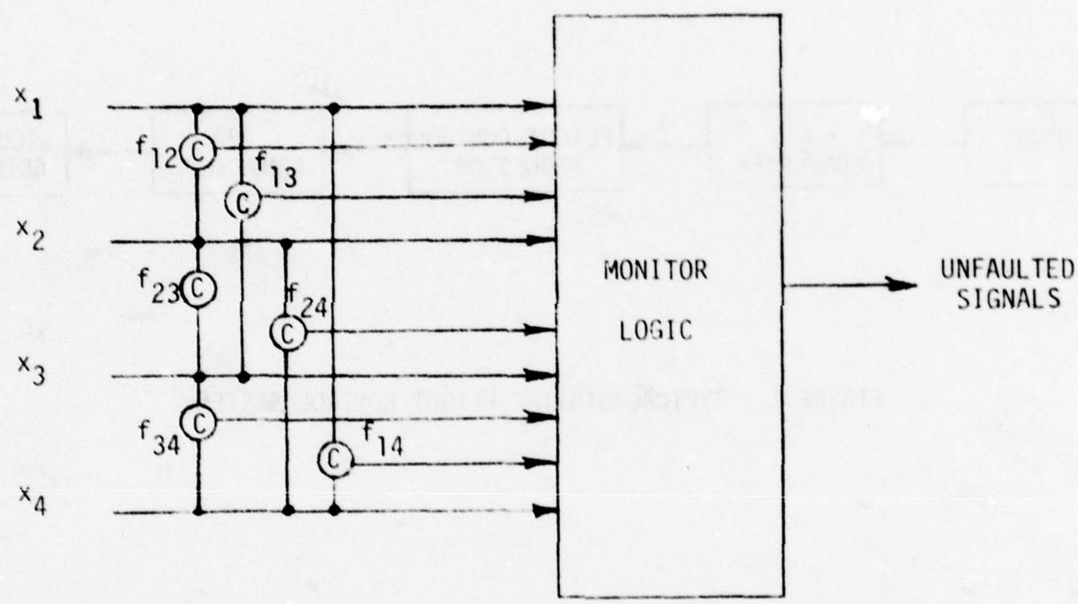


FIGURE 2. INPUT-INPUT COMPARISON MONITORING

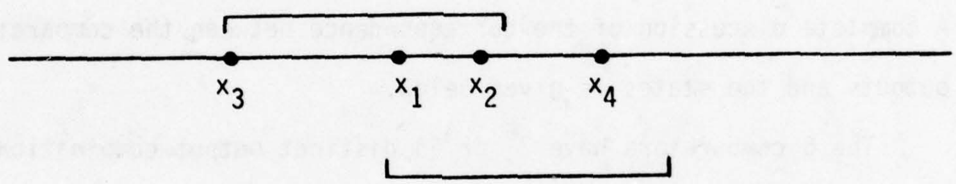


Figure 3 Representation of Signal Values and Tolerance

each other. A simplified representation for the signal relationships in Figure 3 would be:

$$\overline{x_3\underline{x_1 x_2}}\underline{x_4}$$

We call any such set of relationships a state.

When only comparison monitors are used, their outputs cannot be associated with a unique state, but could be produced by several possible states. For our example the 6 comparators produce the outputs $f_{12}=f_{13}=f_{14}=f_{23}=f_{24}=0$, $f_{34}=1$, but these outputs could also be produced by any one of the following three states:

$$\overline{x_3\underline{x_2 x_1}}\underline{x_4} \qquad \overline{x_4\underline{x_1 x_2}}\underline{x_3} \qquad \overline{x_4\underline{x_2 x_1}}\underline{x_3}$$

A complete discussion of the correspondence between the comparator outputs and the states is given below.

The 6 comparators have 2^6 or 64 distinct *output combinations*; however 7 combinations do not occur. Table 1 shows a complete listing of all the combinations that are possible, along with the orderings of $x_1x_2x_3x_4$ that can occur with each combination. There are thus 57 output combinations and 336 states.

An examination of the table shows that only a small number of patterns occur, provided that signal identification numbers are ignored. This fact suggests the categorization given in Table 2. Here, the circles could be any of the four inputs and the bars again show which of the inputs are within tolerance of each other. At fault level 1 the pattern indicates a so-called "soft fault" in that the lowest-valued and the highest-valued signals are out-of-tolerance with each other but in-tolerance with their other neighbors. At level

Nb. Faults	f_{12}	f_{13}	f_{14}	f_{23}	f_{24}	f_{34}	States
0	0	0	0	0	0	0	$\overline{1234}$ $\overline{2134}$ $\overline{3214}$ $\overline{4231}$
							$\overline{1324}$ ... (24 possibilities)
1	1	0	0	0	0	0	$\overline{1\underline{3}}\underline{4}2$ $\overline{14}\underline{32}$ $\overline{2\underline{3}}\underline{4}1$ $\overline{24}\underline{31}$
1	0	1	0	0	0	0	
1	0	0	1	0	0	0	.
1	0	0	0	1	0	0	. .
1	0	0	0	0	1	0	
1	0	0	0	0	0	1	$\overline{3\underline{12}}\underline{4}$ $\overline{3\underline{21}}\underline{4}$ $\overline{4\underline{12}}\underline{3}$ $\overline{4\underline{21}}\underline{3}$
2	1	1	0	0	0	0	$\overline{14}\underline{23}$ $\overline{14}\underline{32}$ $\overline{2\underline{3}}\underline{41}$ $\overline{3\underline{2}}\underline{41}$
2	1	0	1	0	0	0	
2	0	1	1	0	0	0	
2	1	0	0	1	0	0	
2	1	0	0	0	1	0	.
2	0	0	0	1	1	0	. .
2	0	1	0	1	0	0	
2	0	1	0	0	0	1	
2	0	0	0	1	0	1	
2	0	0	1	0	1	0	
2	0	0	1	0	0	1	
2	0	0	0	0	1	1	$\overline{4\underline{1}}\underline{23}$ $\overline{4\underline{1}}\underline{32}$ $\overline{2\underline{3}}\underline{14}$ $\overline{3\underline{2}}\underline{14}$
	0	0	1	1	0	0	DO
	0	1	0	0	1	0	NOT
	1	0	0	0	0	1	OCCUR

3	1	0	0	1	1	0	$1\text{-}2\overline{34}$	$1\text{-}\overline{243}$	$1\text{-}\overline{324}$	$1\text{-}3\overline{42}$
							$1\text{-}\overline{423}$	$1\text{-}\overline{432}$	$\overline{234}\text{-}1$	$\overline{243}\text{-}1$
							$\overline{324}\text{-}1$	$\overline{342}\text{-}1$	$\overline{423}\text{-}1$	$\overline{432}\text{-}1$
3	1	0	0	1	1	0		⋮		
3	0	1	0	1	0	1				
3	0	0	1	0	1	1	$4\text{-}\overline{123}$	$4\text{-}\overline{132}$	$4\text{-}\overline{213}$	$4\text{-}\overline{231}$
							$4\text{-}\overline{312}$	$4\text{-}\overline{321}$	$\overline{123}\text{-}4$	$\overline{132}\text{-}4$
							$\overline{213}\text{-}4$	$\overline{231}\text{-}4$	$\overline{312}\text{-}4$	$\overline{321}\text{-}4$
	0	0	0	1	1	1	DO			
	0	1	1	0	0	1	NOT			
	1	0	1	0	1	0		OCCUR		
	1	1	0	1	0	0				
3	1	1	0	0	1	0	$\overline{2}\underline{34}\overline{1}$	$\overline{1}\underline{43}\overline{2}$		
3	1	1	0	0	0	1				
3	1	0	1	1	0	0				
3	1	0	1	0	0	1		⋮		
3	1	0	0	1	0	1				
3	1	0	0	0	1	1				
3	0	1	1	1	0	0				
3	0	1	1	0	1	0				
3	0	1	0	1	1	0				
3	0	1	0	0	1	1				
3	0	0	1	1	1	0				
3	0	0	1	1	0	1	$\overline{3}\underline{12}\overline{4}$	$\overline{4}\underline{21}\overline{3}$		

4	0	1	1	1	1	0	$\overline{12}$-$\overline{34}$	$\overline{12}$-$\overline{43}$	$\overline{21}$-$\overline{34}$	$\overline{21}$-$\overline{43}$
							$\overline{34}$-$\overline{12}$	$\overline{34}$-$\overline{21}$	$\overline{43}$-$\overline{12}$	$\overline{43}$-$\overline{21}$
4	1	0	1	1	0	1			⋮	
4	1	1	0	0	1	1	$\overline{14}$-$\overline{23}$	$\overline{14}$-$\overline{32}$	$\overline{41}$-$\overline{23}$	$\overline{41}$-$\overline{32}$
							$\overline{23}$-$\overline{14}$	$\overline{23}$-$\overline{41}$	$\overline{32}$-$\overline{14}$	$\overline{32}$-$\overline{41}$
4	1	1	1	1	0	0	1-$\overline{2}4\underline{3}$	1-$\overline{3}4\underline{2}$	$\overline{2}4\underline{3}$-1	$\overline{3}4\underline{2}$-1
4	1	1	1	0	1	0				
4	1	1	1	0	0	1				
4	1	0	0	1	1	1				
4	1	0	1	1	1	0			⋮	
4	1	1	0	1	1	0				
4	0	1	0	1	1	1				
4	0	1	1	1	0	1				
4	1	1	0	1	0	1				
4	0	0	1	1	1	1				
4	0	1	1	0	1	1				
4	1	0	1	0	1	1	4-$\overline{1}3\underline{2}$	4-$\overline{2}3\underline{1}$	$\overline{1}3\underline{2}$-4	$\overline{2}3\underline{1}$-4
5	1	1	1	1	1	0	1-2-$\overline{34}$	1-2-$\overline{43}$	2-1-$\overline{34}$	2-1-$\overline{43}$
							1-$\overline{34}$-2	1-$\overline{43}$-2	2-$\overline{34}$-1	2-$\overline{43}$-1
							$\overline{34}$-1-2	$\overline{43}$-1-2	$\overline{34}$-2-1	$\overline{43}$-2-1
5	1	1	1	1	0	1				
5	1	1	1	0	1	1			⋮	
5	1	1	0	1	1	1				
5	1	0	1	1	1	1				

5	0	1	1	1	1	1	3-4-$\overline{12}$	3-4-$\overline{21}$	4-3-$\overline{12}$	4-3-$\overline{21}$
							3-$\overline{12}$-4	3-$\overline{21}$-4	4-$\overline{12}$-3	4-$\overline{21}$-3
							$\overline{12}$-3-4	$\overline{21}$-3-4	$\overline{12}$-4-3	$\overline{21}$-4-3
6	1	1	1	1	1	1	1-2-3-4	2-1-3-4	3-2-1-4	4-2-3-1
							etc. (24 possibilities)			

Note: 1234 denotes $x_1x_2x_3x_4$, etc.

Table 1 All Possible Fault Combinations

Fault Level	Pattern	Description
0	$\overline{0000}$	No Faults
1	$\overline{00\underline{0}}\underline{0}$	Soft Fault
2A	$\overline{0}0\underline{00}$	Soft Fault
2B	$\overline{000}\underline{0}$	Soft Fault
3A	$0\text{-}\overline{000}$	Hard Fault
3B	$\overline{0}\underline{0}\overline{0}0$	Soft Fault
3C	$\overline{000}\text{-}0$	Hard Fault
4A	$0\text{-}0\overline{0}\underline{0}$	Hard and Soft Fault
4B	$\overline{00}\text{-}\overline{00}$	Split Fault
4C	$\overline{0}\underline{0}0\text{-}0$	Hard and Soft Fault
5A	$0\text{-}0\text{-}\overline{00}$	Double Hard Fault
5B	$0\text{-}\overline{00}\text{-}0$	Double Hard Fault
5C	$\overline{0}0\text{-}0\text{-}0$	Double Hard Fault
6	0-0-0-0	Four Hard Faults

Table 2 Fourteen Fault Patterns

3A the pattern indicates a "hard fault" in that the lowest-valued signal is out-of-tolerance with all three remaining signals.

The use of patterns reduces the number of possible relationships from 336 states to 14 patterns. However, it omits some of the information supplied by the comparison monitors. For example, the pattern $\overline{\bigcirc\underline{\bigcirc\bigcirc}}\underline{\bigcirc}$ occurs anytime there is a single soft fault, but this corresponds to any of 6 combinations of outputs from the comparison monitors and to 24 states.

There are 24 possible ways to order the 4 signals; hence for 14 patterns, we again arrive at 24 x 14 or 336 states. Table 3 shows these 24 orderings of the signals and the comparator outputs for each of the 14 patterns. This table shows the correspondence between a given ordering of signals and the fault level, the pattern, and the comparator outputs.

2.3 Keys

The state may change each time one or more of the signals change. Likewise, the pattern may change, as we discuss next.

Assume that the set of four signals is examined by the monitoring algorithm each time any one of the signals is updated. (In those situations where more than one signal could change at the same time, we could still process all four after each signal change.) With this restriction it is possible to develop groups of patterns, or Keys, which consist of all patterns to which the present pattern may move after a change in one signal. Note that the Keys do not allow us to trace the sequence of states that occur--only the patterns.

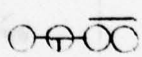
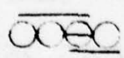
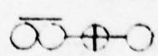
Signal ordering	Level, Patterns and Comparator Outputs						
	0:$\overline{0000}$	1:$\overline{00}00$	2A:$\overline{0}\underline{000}$	2B:$0\overline{0\underline{0}}\underline{0}$	3A:0-$\overline{000}$	3B:$\overline{000}$-0	3C:$\overline{0}\underline{0}\overline{\underline{0}}\overline{0}$
1234	000000	001000	011000	001001	111000	001011	011010
1243	000000	010000	011000	010100	111000	010101	011100
1324	000000	001000	101000	001001	111000	001011	101001
1342	000000	100000	101000	100100	111000	100110	101100
1423	000000	010000	110000	010001	111000	010101	110001
1432	000000	100000	110000	100010	111000	100110	110010
2134	000000	000010	000110	001010	100110	001011	001110
2143	000000	000100	000110	010100	100110	010101	010110
2314	000000	000010	100010	000011	100110	001011	100011
2341	000000	100000	100010	110000	100110	111000	110010
2413	000000	000100	100100	000101	100110	010101	100101
2431	000000	100000	100100	101000	100110	111000	101100
3124	000000	000001	000101	001001	010101	001011	001101
3142	000000	000100	000101	100100	010101	100110	100101
3214	000000	000001	010001	000011	010101	001011	010011
3241	000000	010000	010001	110000	010101	111000	110001
3412	000000	000100	010100	000110	010101	100110	010110
3421	000000	010000	010100	011000	010101	111000	011100
4123	000000	000001	000011	010001	001011	010101	010011
4132	000000	000010	000011	100010	001011	100110	100011
4213	000000	000001	001001	000101	001011	010101	001101
4231	000000	001000	001001	101000	001011	111000	101001
4312	000000	000010	001010	000110	001011	100110	001110
4321	000000	001000	001010	011000	001011	111000	011010

Table 3 Signal Ordering and Fault Level, Patterns, and Comparator Outputs

Signal ordering	Level, Patterns and Comparator Outputs						
	4A:0-$\overline{000}$	4C:$\overline{000}$-0	4B:$\overline{00}$-$\overline{00}$	5A:0-0-$\overline{00}$	5B:0-$\overline{00}$-0	5C:$\overline{00}$-0-0	6:0-0-0-0
1234	111010	011011	011110	111110	111011	011111	111111
1243	111100	011101	011110	111110	111101	011111	111111
1324	111001	101011	101101	111101	111011	101111	111111
1342	111100	101110	101101	111101	111110	101111	111111
1423	111001	110101	110011	111011	111101	110111	111111
1432	111010	110110	110011	111011	111110	110111	111111
2143	101110	001111	011110	111110	101111	011111	111111
2134	110110	010111	011110	111110	110111	011111	111111
2314	100111	101011	110011	110111	101111	111011	111111
2341	110110	111010	110011	110111	111110	111011	111111
2413	100111	110101	101101	101111	110111	111101	111111
2431	101110	111100	101101	101111	111110	111101	111111
3124	011101	001111	101101	111101	011111	101111	111111
3142	110101	100111	101101	111101	110111	101111	111111
3214	010111	011011	110011	110111	011111	111011	111111
3241	110101	111001	110011	110111	111101	111011	111111
3412	010111	110110	011110	011111	110111	111110	111111
3421	011101	111100	011110	011111	111101	111110	111111
4123	011011	010111	110011	111011	011111	110111	111111
4132	101011	100111	110011	111011	101111	110111	111111
4213	001111	011101	101101	101111	011111	111101	111111
4231	101011	111001	101101	101111	111011	111101	111111
4312	001111	101110	011110	011111	101111	111110	111111
4321	011011	111010	011110	011111	111011	111110	111111

Table 3 Signal Ordering and Fault Level, Patterns, and Comparator Outputs (Continued)

There are 7 basic Keys, as shown in Figure 4. Key A is one of the simplest and most important. For this Key the symbol ● indicates a signal which, once it is updated, can then occupy any other position marked by ● and thus produce a new state having a pattern within the same Key. The symbol ○ indicates a signal that, when it changes, produces a state with a pattern outside of the Key. For example, when the lowest-valued signal in the pattern  of Key A is updated, the new pattern could be unchanged or it could be any of the other 6 patterns in Key A; the new pattern is guaranteed to be in Key A, provided that the given signal was the one updated. However, if any of the signals denoted by ○ is the next one to change, the new state will either remain the same or change to a state *having a pattern in one of the other Keys--but not Key A.*

There is some overlap between Keys C and E, and also between Keys D and F. For Keys C and E consider the pattern  . If the second-lowest signal moves to be left there are two possible results: either  or  , which are in different Keys (C and E, respectively). Similarly, for the pattern  in Keys D and F, if the second-highest signal moves to the right, this could lead either to  or to  . In these two cases, which next pattern occurs depends on the magnitude of the hard fault that had been present and not just the fact that the signal differences exceeded the specified tolerance. Thus, although Keys C and E (D and F) are distinct, they cannot be distinguished with only the information supplied by the comparators.

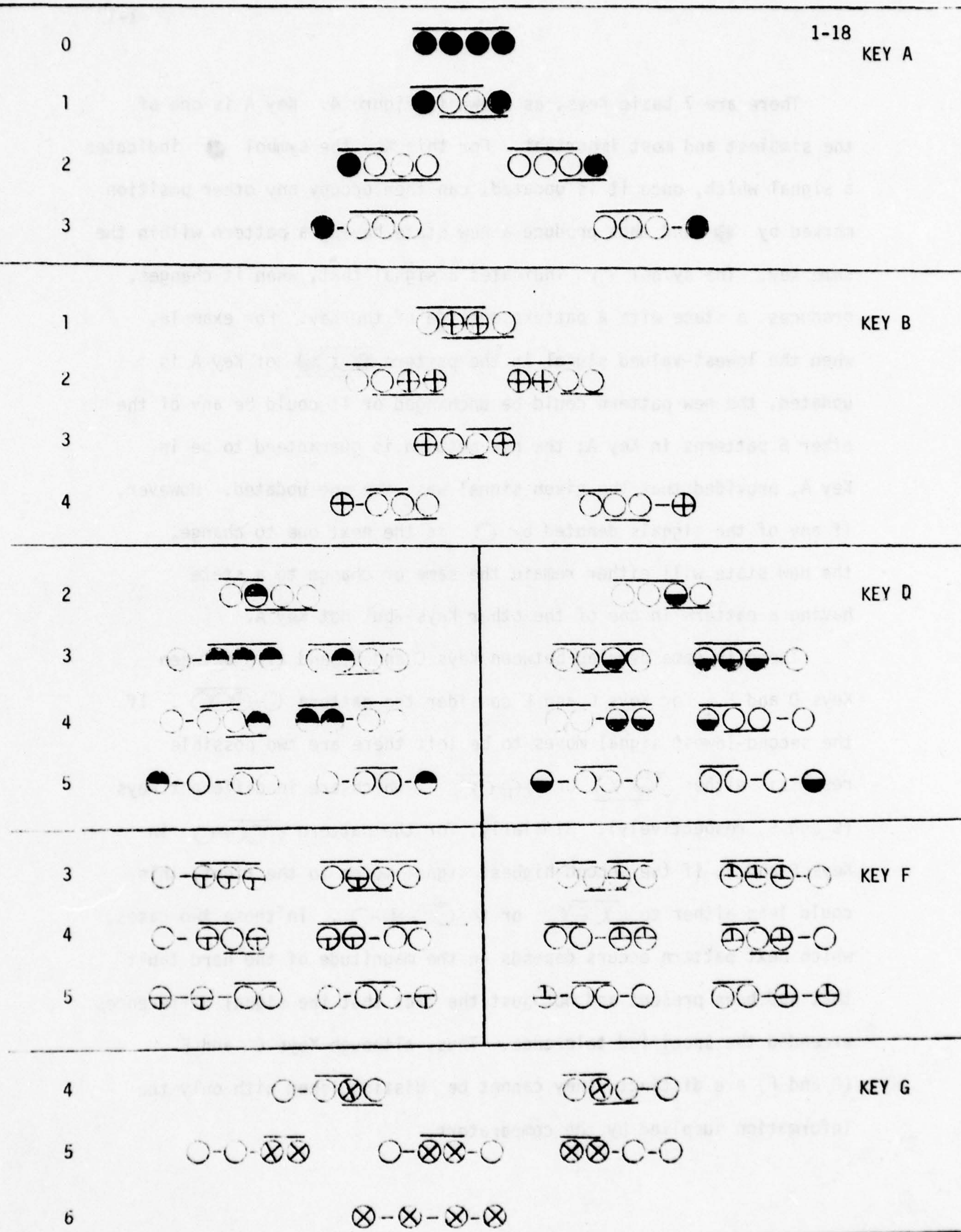
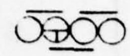


FIGURE 4 SEVEN KEYS OF MOVEMENT

Another way to present the information of Figure 4 is shown in Table 4. This representation clearly shows the ambiguity present at fault levels 3, 4, and 5. As the number of hard faults increases, the inability of the comparators to indicate fault magnitudes causes additional overlapping of Keys. For example, strictly speaking we cannot move from  (Level 5, Key E) to  (Level 3, Key E). However, we will not enumerate additional Keys because they are not used in the monitoring algorithms presented.

Pattern	Position Changed 1	2	3	4
0	A	A	A	A
1	A	B	B	A
2A	A	C	B	B
2B	B	B	D	A
3A	A	C,E	C,F	C,E
3B	B	C,E	D,F	B
3C	D,F	D,F	D,F	A
4A	B	E	G	C,E
4B	C,E	C,E	D,F	D,F
4C	D,F	G	F	B
5A	C,E	E	G	G
5B	D,F	G	G	C,E
5C	G	G	F	D,F
6	G	G	G	G

Table 4 Allowable Pattern Movements

Within and Among Keys

3.0 BASIC ALGORITHM FOR MONITORING

3.1 Relationship to the Signal Characterization

In this section we present two algorithms for comparison monitoring based on the characterization of the previous section. In the first algorithm the basic idea is to consider all four signals to be good as long as the fault level is 0 or 1, but to eliminate one signal at fault levels 2, 3, or 4. With these restrictions fault levels 5 and 6 are not needed because their patterns are not obtainable. Furthermore, only Keys A and B are needed since, from fault levels 0 and 1, they contain the only possible routes to the other states at fault levels 2, 3, and 4, at which levels a signal is eliminated and the redundancy is reduced from 4 to 3.

Figure 5 shows the combined Keys A and B for the first algorithm. Once a signal is eliminated, the tri-redundant case becomes applicable (Figure 6). In the tri-redundant case we again consider the signals as good for fault levels 0 and 1 and eliminate the hard-faulted signal at levels 2 and 3.

Assume that at system start-up the signal pattern is at fault level 0 or 1. If the pattern stays at these levels, no signals are eliminated. If the pattern moves to fault level 3 or 4 then the comparison monitor should declare the most recently updated signal as being faulted. If the pattern moves to fault level 2 the comparison monitor will still select one signal as being faulted but the logic for identifying the faulted signal is more complicated and will be treated below during the discussion of the flowchart for the algorithm.

The second algorithm is simpler than the first and will be treated only briefly. The simplicity arises from the constraint that

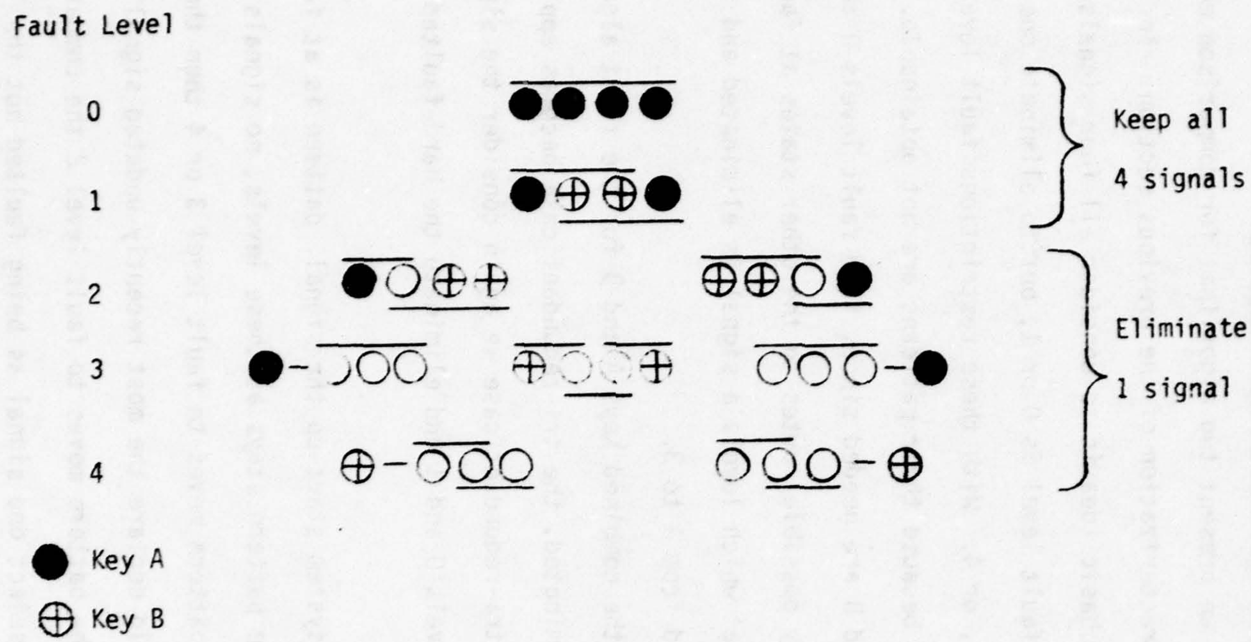


Figure 5 Combined Keys A and B, First Algorithm

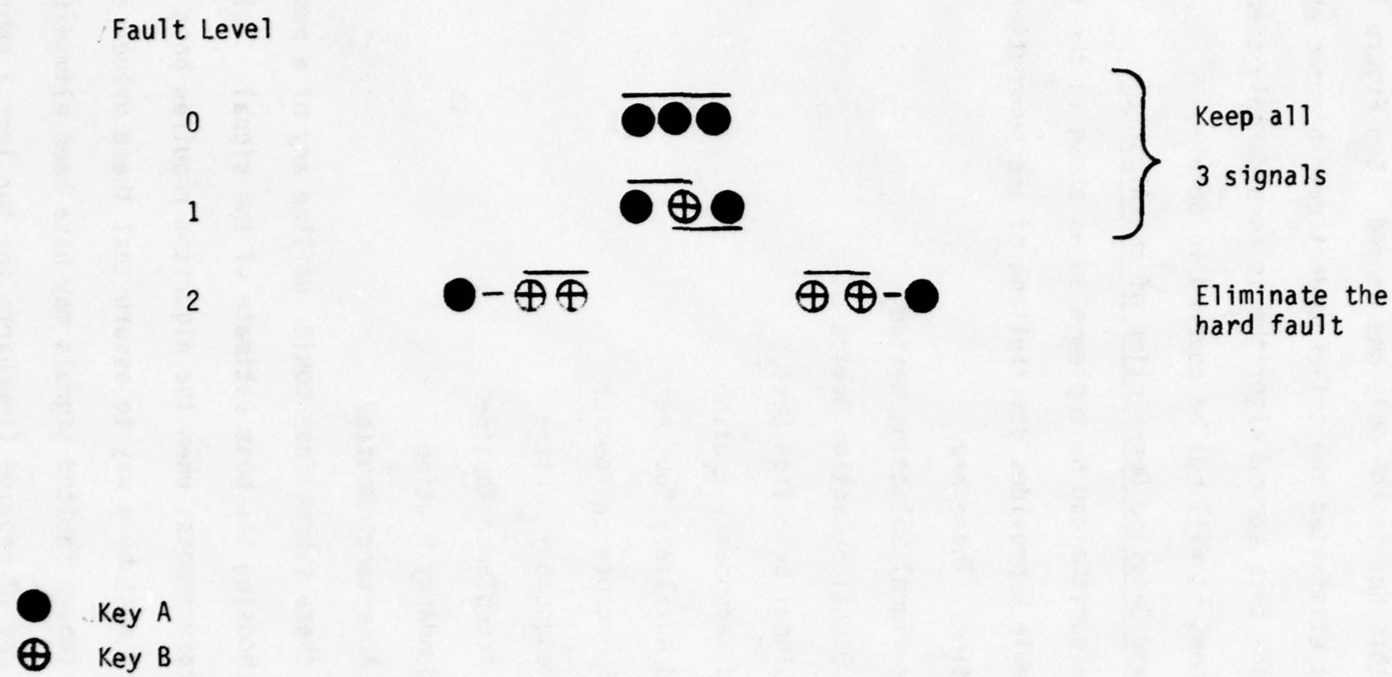


Figure 6 Combined Keys A and B for the Tri-redundant Case, First Algorithm

a signal is declared faulted and eliminated when the pattern is at any fault level other than 0. Since Key A contains the pattern at fault level 0, this Key is the only one needed. See Figure 7. Once a signal is eliminated the tri-redundant case becomes applicable (Figure 8). Since this second algorithm is considerably simpler than the first one, it will not be covered in detail.

3.2 Flowchart and Detailed Description of the Algorithm

The first algorithm can be implemented as shown in the flowchart of Figure 9. Table 5 provides descriptions of the subroutines used by the main routine. These are

QSSD: Quad Signal Selection Device

TSSD: Tri Signal Selection Device

BSSD: Bi Signal Selection Device

QUAD: Quad Redundancy Routine

QAUX: Quad Auxiliary Routine

TACPT: Tri Acceptance Routine

TRI: Tri Redundancy Routine

BACP1: Bi Acceptance Routine

BI: Bi Redundancy Routine

BACP2: Bi Acceptance Routine

The first 3 of these subroutines could utilize any of a number of algorithms for choosing the best estimate of the signal value based on redundant measurements; when the algorithm requires past values of the signals there must be a way to assure that these values are available, even though faulted signals may have been eliminated. Figures 10, 11, and 12 provide flowcharts for the last 7 subroutines used by the first algorithm.

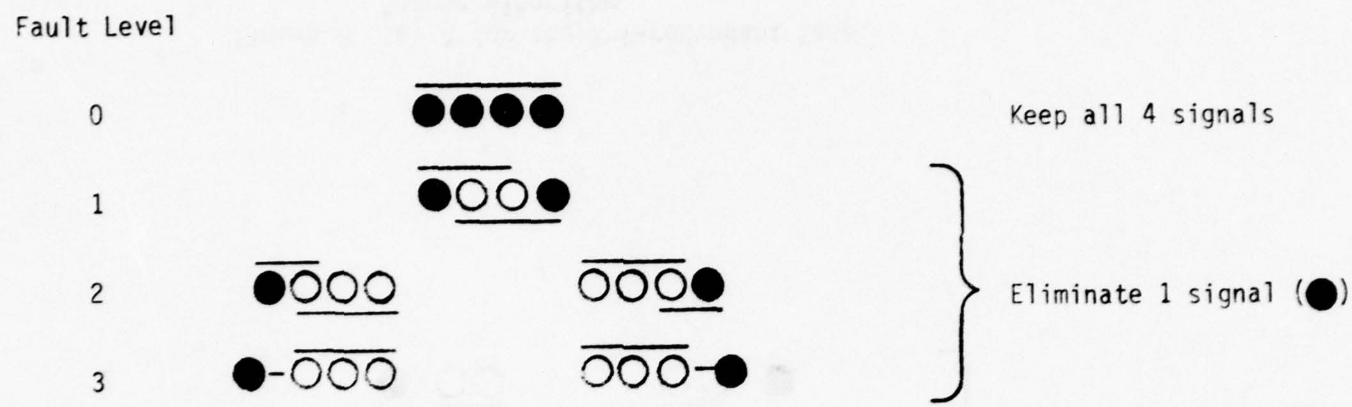


Figure 7 Key A, Second Algorithm

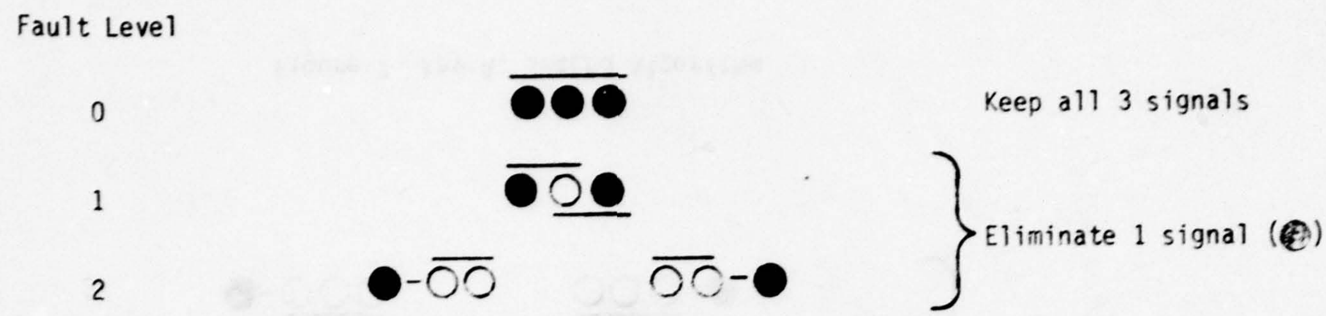


Figure 8 Key A for the Tri-redundant Case,
Second Algorithm

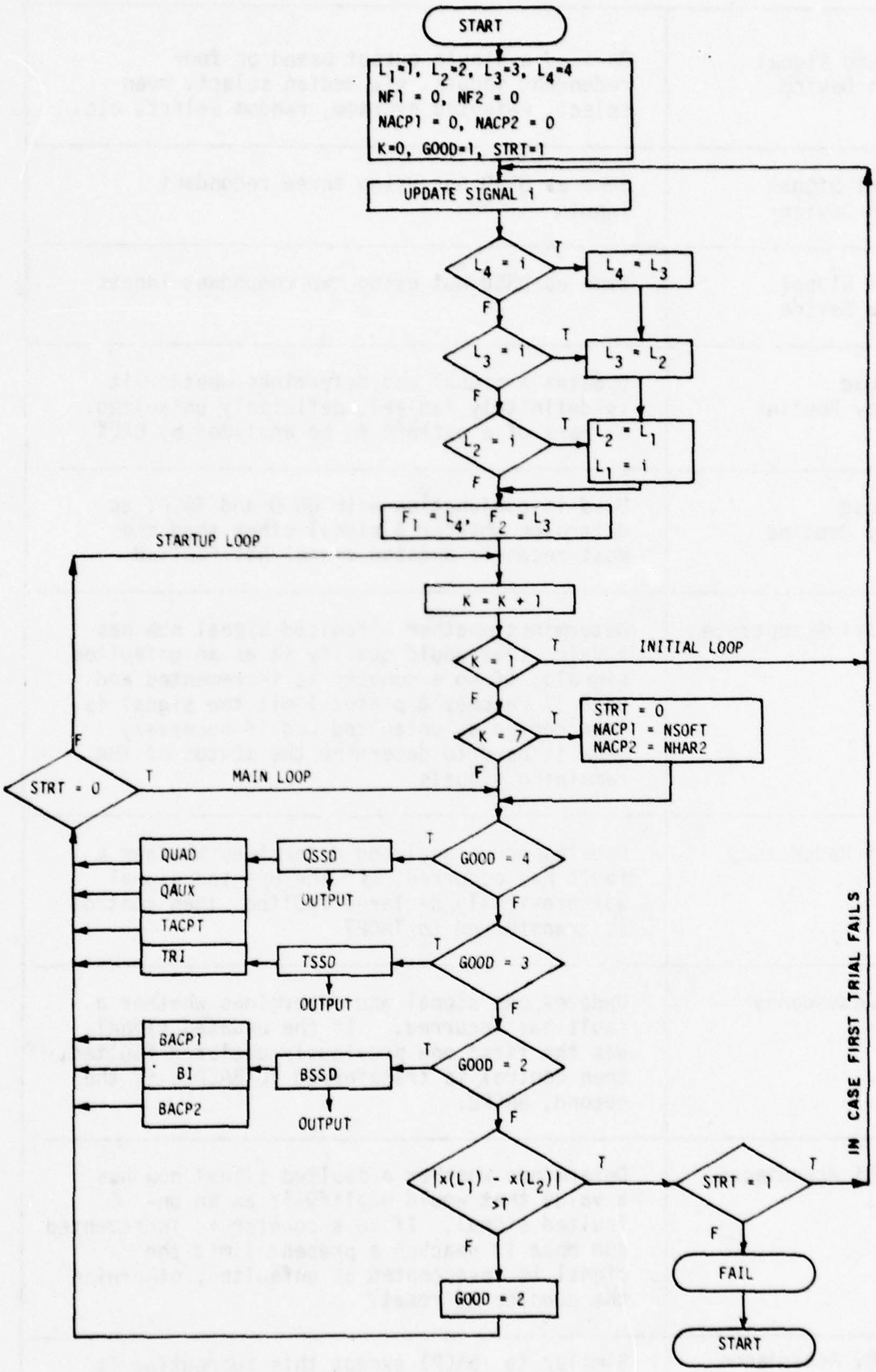


FIGURE 9

OVERALL FLOWCHART – COMBINED STARTUP AND MAIN ROUTINE.

Table 5 Subroutines Used by the First Algorithm

QSSD: Quad Signal Selection Device	Devises a single output based on four redundant inputs, via median select, mean select, weighted average, random select, etc.
TSSD: Tri Signal Selection Device	Same as QSSD but using three redundant inputs
BSSD: Bi Signal Selection Device	Same as BSSD but using two redundant inputs
QUAD: Quad Redundancy Routine	Updates a signal and determines whether it is definitely faulted, definitely unfaulted, or part of a pattern to be analyzed by QAUX
QAUX: Quad Auxiliary Routine	Used in conjunction with QUAD and TACPT to determine whether a signal other than the most recently updated signal has faulted
TACPT: Tri Acceptance Routine	Determines whether a faulted signal now has a value that would qualify it as an unfaulted signal. If so a counter is incremented and once it reaches a preset limit the signal is re-accepted as unfaulted and if necessary QAUX is used to determine the status of the remaining signals
TRI: Tri Redundancy Routine	Updates one signal and determines whether a fault has occurred. If the updated signal was previously declared faulted, then control is transferred to TACPT.
BI: Bi Redundancy Routine	Updates one signal and determines whether a fault has occurred. If the updated signal was the first one previously declared faulted, then control is transferred to BACP1; if the second, BACP2.
BACP1: Bi Acceptance Routine 1	Determines whether a faulted signal now has a value that would qualify it as an unfaulted signal. If so a counter is incremented and once it reaches a present limit the signal is re-accepted as unfaulted; otherwise the counter is reset.
BACP2: Bi Acceptance Routine 2	Similar to BACP1 except this subroutine is for the second failed signal.

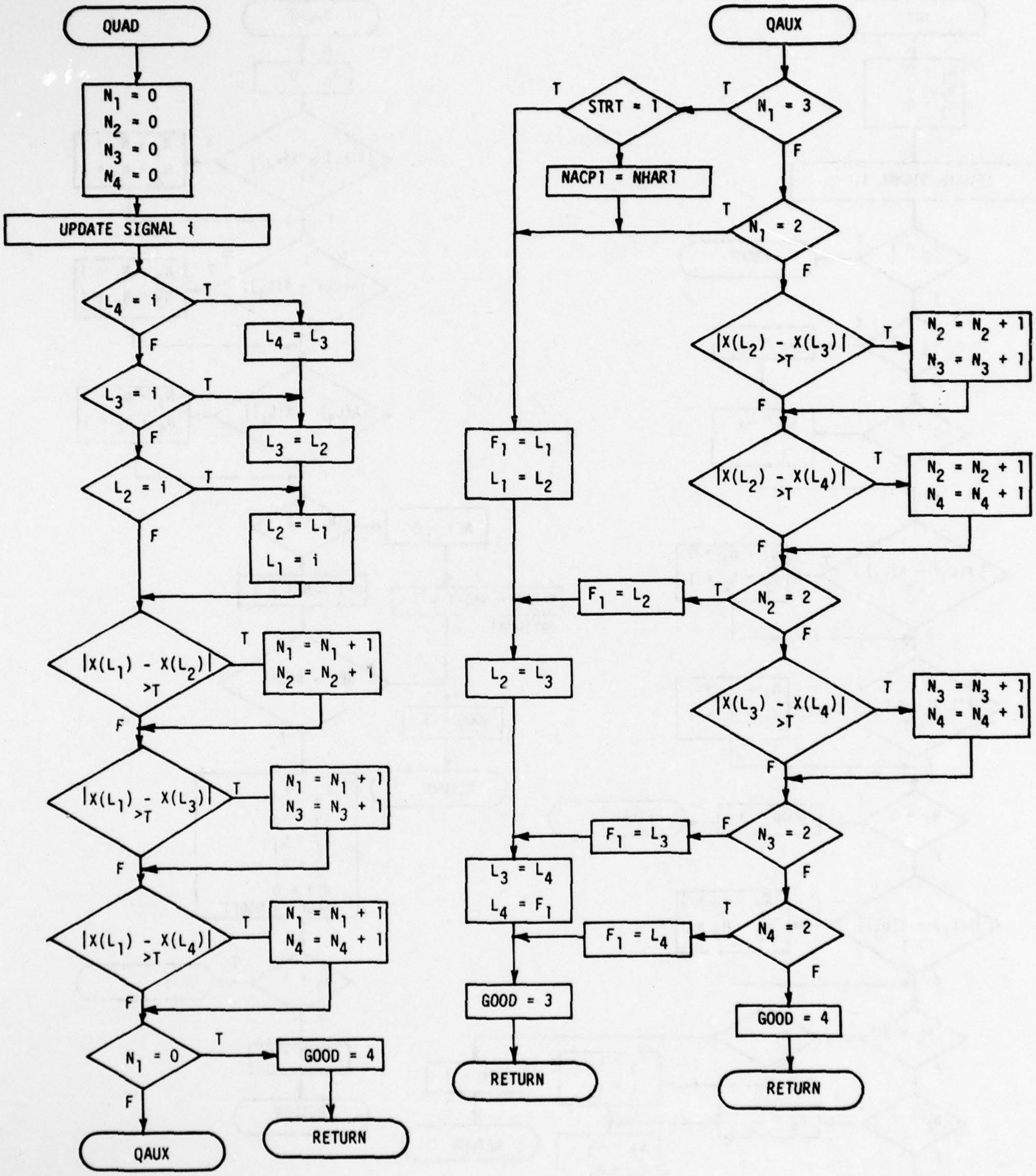


FIGURE 10

QUAD AND QAUX FAULT DETECTION SUBROUTINES.

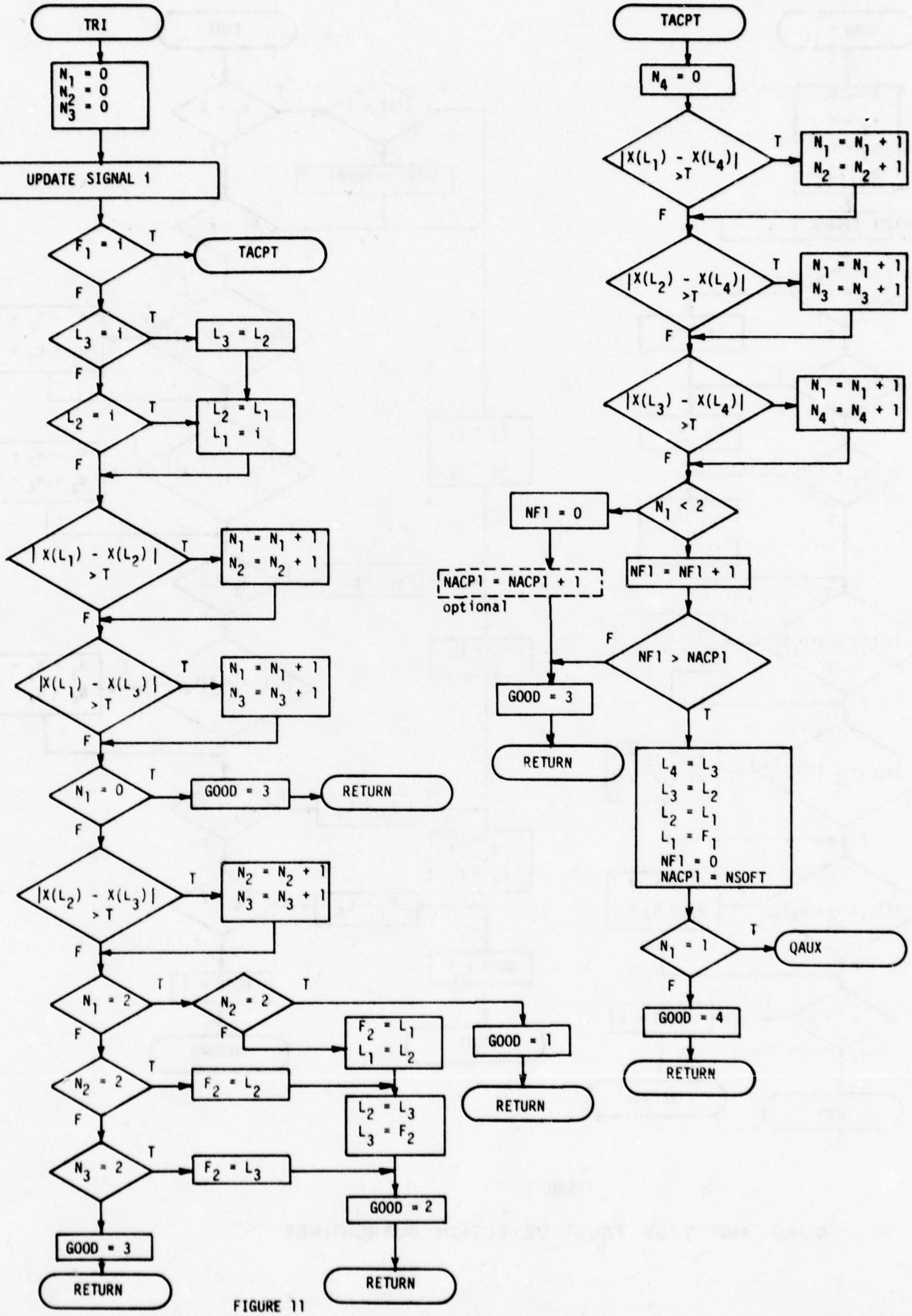


FIGURE 11

TRI AND TACPT ROUTINES.

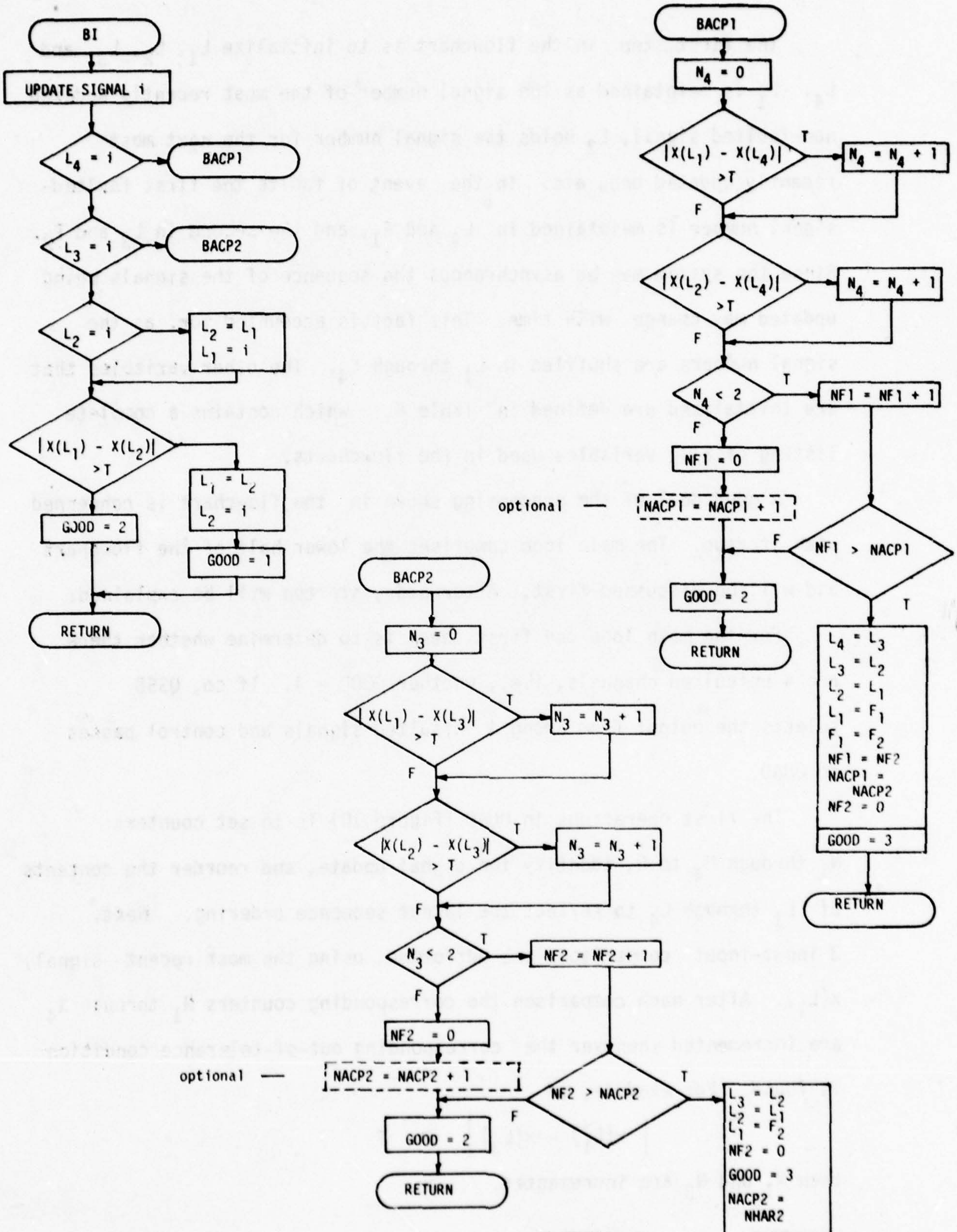


FIGURE 12

BI ERROR DETECTION AND ACCEPTING ROUTINES.

The first step in the flowchart is to initialize L_1, L_2, L_3, and L_4. L_1 is maintained as the signal number* of the most recently updated non-faulted signal, L_2 holds the signal number for the next most recently updated one, etc. In the event of faults the first faulted-signal number is maintained in L_4 and F_1, and the second in L_3 and F_2. Since the system may be asynchronous the sequence of the signals being updated may change with time. This fact is accounted for, as the signal numbers are shuffled in L_1 through L_4. The other variables that are initialized are defined in Table 6, which contains a complete listing of all variables used in the flowcharts.

A good deal of the processing shown in the flowchart is concerned with startup. The main loop comprises the lower half of the flowchart and will be discussed first. Afterwards, startup will be explained.

For the main loop the first check is to determine whether there are 4 unfaulted channels, i.e., whether GOOD = 4. If so, QSSD selects the output from among 4 unfaulted signals and control passes to QUAD.

The first operations in QUAD (Figure 10) is to set counters N_1 through N_4 to 0, identify the signal update, and reorder the contents of L_1 through L_4 to reflect the latest sequence ordering. Next, 3 input-input comparisons are performed, using the most recent signal, $x(L_1)$. After each comparison the corresponding counters N_1 through N_4 are incremented whenever the corresponding out-of-tolerance condition is found. For example, if

$$\left| x(L_1) - x(L_3) \right| > T$$

then N_1 and N_3 are incremented.

---

*The signals may be thought of as being numbered 1 through 4.

Variable	Definition
L_1	Signal number of the most recently updated non-faulted signal
L_2	Signal number of the next most recently updated non-faulted signal
L_3	Signal number of the third most recently updated non-faulted signal, or the signal number of the second faulted signal
L_4	Signal number of the fourth most recently updated non-faulted signal, or the signal number of the first faulted signal
F_1	Same as L_4 for GOOD $<$ 4
F_2	Same as L_3 for GOOD $<$ 3
NF1	Counter for the number of times faulted signal F1 qualified as unfaulted
NF2	Same as NF1 except for faulted signal F2
NACP1	Maximum count for NF1 before faulted signal is re-accepted
NACP2	Same as NF2 except for NF2
K	Counter used in start-up
GOOD	Specifies the number of unfaulted signals
STRT	Flag
NSOFT	Value of NACP1 or NACP2 for a soft fault
NHAR1	Value of NACP1 or NACP2 for a hard fault
NHAR2	Value of NACP1 or NACP2 for a hard fault
N_1	Comparison-error counter for $x(L_1)$
N_2	Comparison-error counter for $x(L_2)$
N_3	Comparison-error counter for $x(L_3)$
N_4	Comparison-error counter for $x(L_4)$

Table 6 Variables Used in the Flowcharts

Next, N_1 is tested for a fault in the most recently updated signal. To see how this works consider the examples shown in Figure 13, in which the signal being updated is L_1 and the values of N_1, N_2, N_3, and N_4 are given after the 3 comparisonsshown in Figure 10 are made. Figure 14 shows the counter values, after 3 comparisons, for the most recently updated signals (Key A or Key B). The values below ● and ⊕ correspond to N_1 and those below the unmarked signals are for whatever counter (N_2, N_3, or N_4) corresponds to that signal.

Examination of Figure 14 shows that:

(1) When N_1 is 0, all signals can be declared as unfaulted

(2) When N_1 is 2 or 3, the most recently updated signal can be declared as faulted

(3) When N_1 is 1, either all signals are unfaulted (Key A, fault level 1) or we have a pattern where a fault may or may not exist (Key B, fault level 2).

If the third case is present ($N_1 = 1$) then all 6 comparisons are needed and the subroutine QAUX (Figure 10) is used.* Figure 15 shows the counter values after 6 comparisons are made.

Once a fault is detected in QAUX, F_1 is set equal to the signal number of the faulted signal and then the contents of L_1 through L_4 are changed so that L_4 now corresponds to the first failed channel; then GOOD is set to 3 so that the next time signal selection is performed TSSD, rather than QSSD, will be used.

The operation of TRI is similar to that for QUAD. First, counters N_1 through N_3 are reset and a signal is updated. If that signal is

*QAUX is used any time $N_1 \geq 1$ but only in the case $N_1=1$ are more than 3 comparisons needed.

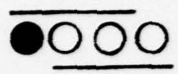
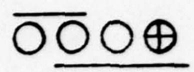
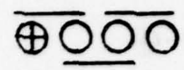
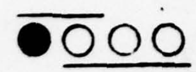
			
$L_1\ L_3\ L_2\ L_4$	$L_4\ L_2\ L_3\ L_1$	$L_1\ L_4\ L_2\ L_3$	$L_1\ L_3\ L_4\ L_2$
$N_1: 0 + 0 + 0 + 1 = 1$	$N_1 = 1$	$N_1 = 2$	$N_1 = 2$
$N_2: 0 + \quad 0 \quad = 0$	$N_2 = 0$	$N_2 = 1$	$N_2 = 1$
$N_3: 0 + 0 \quad = 0$	$N_3 = 1$	$N_3 = 1$	$N_3 = 0$
$N_4: 0 \quad + 1 = 0$	$N_4 = 1$	$N_4 = 0$	$N_4 = 1$

Figure 13 Counter Values After Three Comparisons With the Most Recent Signal, $X(L_1)$

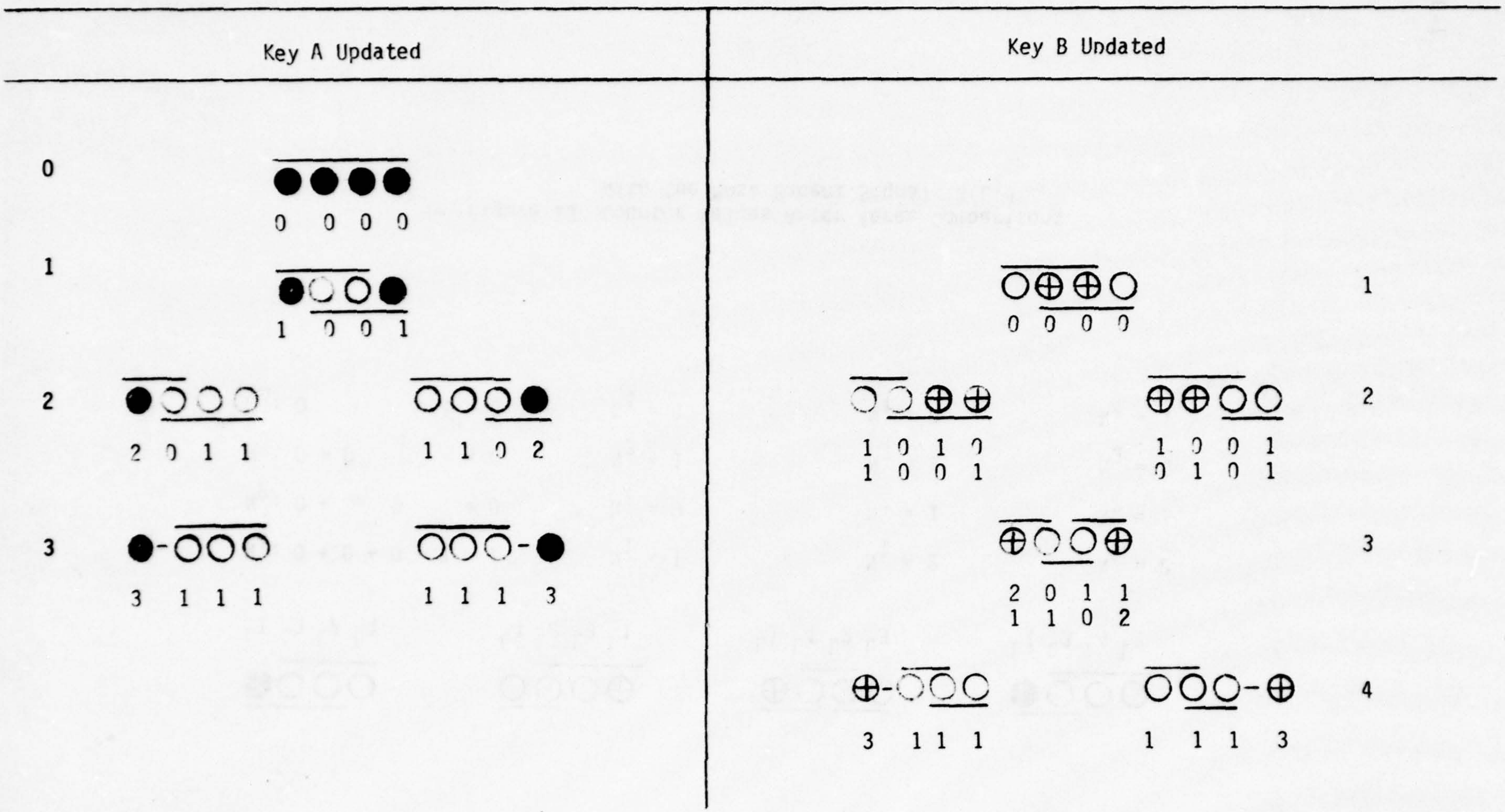


Figure 14 Counter Values After 3 Comparisons with the Most Recently Updated Signal

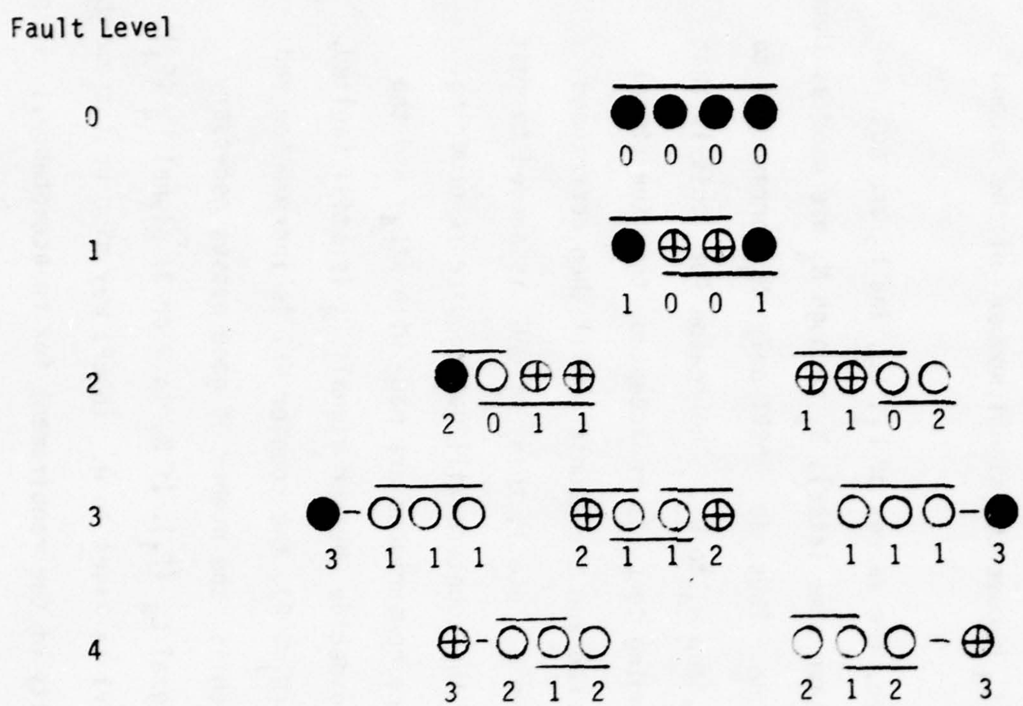


Figure 15 Counter Values After 6 Comparisons with the Most Recently Updated Signal

the one that was previously declared as faulted, then control goes to TACPT to test whether that signal can now be re-accepted as unfaulted. If the updated signal was not previously declared as faulted, then the values of L_1, L_2, and L_3 are re-arranged and the new value is tested for a failure. In the tri-redundant case a failure occurs if any counter reaches 2. (See Figure 16.)

Note that it is possible for all signals to fail. If only 1 failure occurs, F_2 and L_3 become the signal number of the second failed signal.

If TACPT is entered, the values of L_1, L_2, and L_3 are not re-ordered. Instead (to save time later), N_1 through N_4 are used as though the re-ordering were done. Thus, in TACPT only, N_1 corresponds to L_4, N_2 to L_1, N_3 to L_2, and N_4 to L_3. The reason for this is that if the signal corresponding to L_4 is re-accepted, then the signal numbers will be re-ordered and the counters will then correspond exactly (L_1 to N_1, L_2 to N_2, etc.); then if QAUX is needed to test the remaining signals, the counters will not require re-ordering.

Within TACPT, three comparisons are made with $x(L_4)$ and the counter N_1 is tested to decide whether signal L_4 is still faulted. If it is not faulted ($N_1 < 2$), the counter NF1 is incremented and compared to NACP1, which is the number of good passes necessary before re-accepting signal L_4 (F_1). If N_1 is 2 or 3, signal L_4 (F_1) is still faulted and NF1 is reset to 0. (NACP1 may also be incremented to increase the severity of the requirement for re-acceptance.) If signal

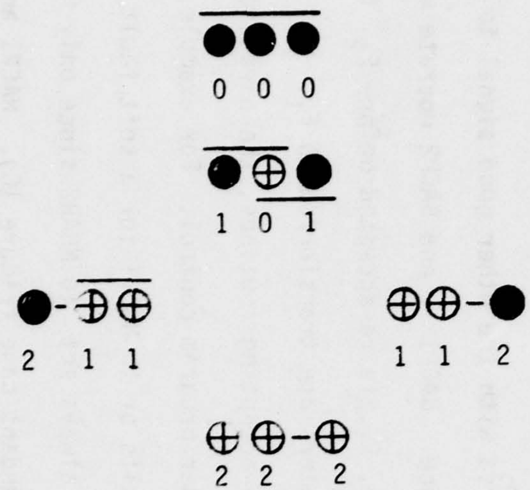


Figure 16 Counter Values After 3 Comparisons with the Most Recently Updated Signal (Tri-Redundant Case)

L_4 is re-accepted (NF1 > NACP1), then the signal numbers are re-ordered to show signal F_1 as the most recent update. Also, if $N_1 = 0$ then we are assured of being at fault level 0 or 1 and we can return with GOOD = 4. Otherwise, $N_1 = 1$ and we must enter QAUX to further test the 4 signals. For example, if either or is the pattern after re-accepting signal , then N_1 will equal 1 and we may or may not have a new failure; QAUX will make this determination.

The BI subroutine receives an updated signal and identifies it. If it is a faulted signal, then either BACP1 or BACP2 is used to determine whether the faulted signal should be re-accepted. Otherwise the updated signal is compared with the other good signal to determine whether it is within tolerance. BACP1 and BACP2 operate much like TACPT. If the first failure, F_1, is re-accepted before F_2, then F_2 and its corresponding parameters are transferred to F_1.

Note that in all the re-accepting routines, the parameters NACP1 and NACP2 can vary under program control. For example, NACP1 is set to NHAR1 for hard faults or to NSOFT for a soft fault. For the second failure NACP2 is always set to NHAR2 since only hard faults occur in the tri-redundant case (Figure 16). NACP1 and NACP2 may also be incremented each time a faulted signal takes on a new, out-of-tolerance value. (This is noted by the optional blocks in Figure 11 and Figure 12.) NACP1 and NACP2 are also used during start-up; here they are initialized to 0 to facilitate a fast re-acceptance of the signals.

This completes the discussion of the main loop and associated

subroutines for the first algorithm. The startup sequence is now treated.

After the variables listed in the first block of Figure 9 are initialized, a signal update is received and the values of L_1 through L_4 are adjusted. Then F_1 is set equal to L_4 and F_2 to L_3. Next, the counter K is incremented and tested. Since this was the first signal updated (K = 1) we loop back to update another signal in the same manner. Once two signals have been updated and their signal numbers sorted, we enter the decision block at the bottom of the flowchart. Then, if these two most recent signals are within tolerance, GOOD is set to 2 and we enter the main portion of the routine with NACP1 and NACP2 both equal to 0. (This facilitates a fast re-acceptance of the signals.) If the two most recent signals are not within tolerance we return to the top of the routine to update another signal and try again to find two signals within tolerance of each other; however, once K becomes equal to 7 the flag STRT is reset and NACP1 and NACP2 are made equal to their nominal values before continuing. Now if GOOD is still 1 (or if GOOD becomes 1 during normal operation) the test associated with the right-most decision block will indicate the FAIL condition and the program will attempt to restart.

The routines for the second algorithm can be obtained by simplifying those for the first algorithm. In the second algorithm both QAUX and the lower portion of TRI can be eliminated, as shown in Figure 17. Note that there are two added parameters, NSOF1 and NSOF2, since now

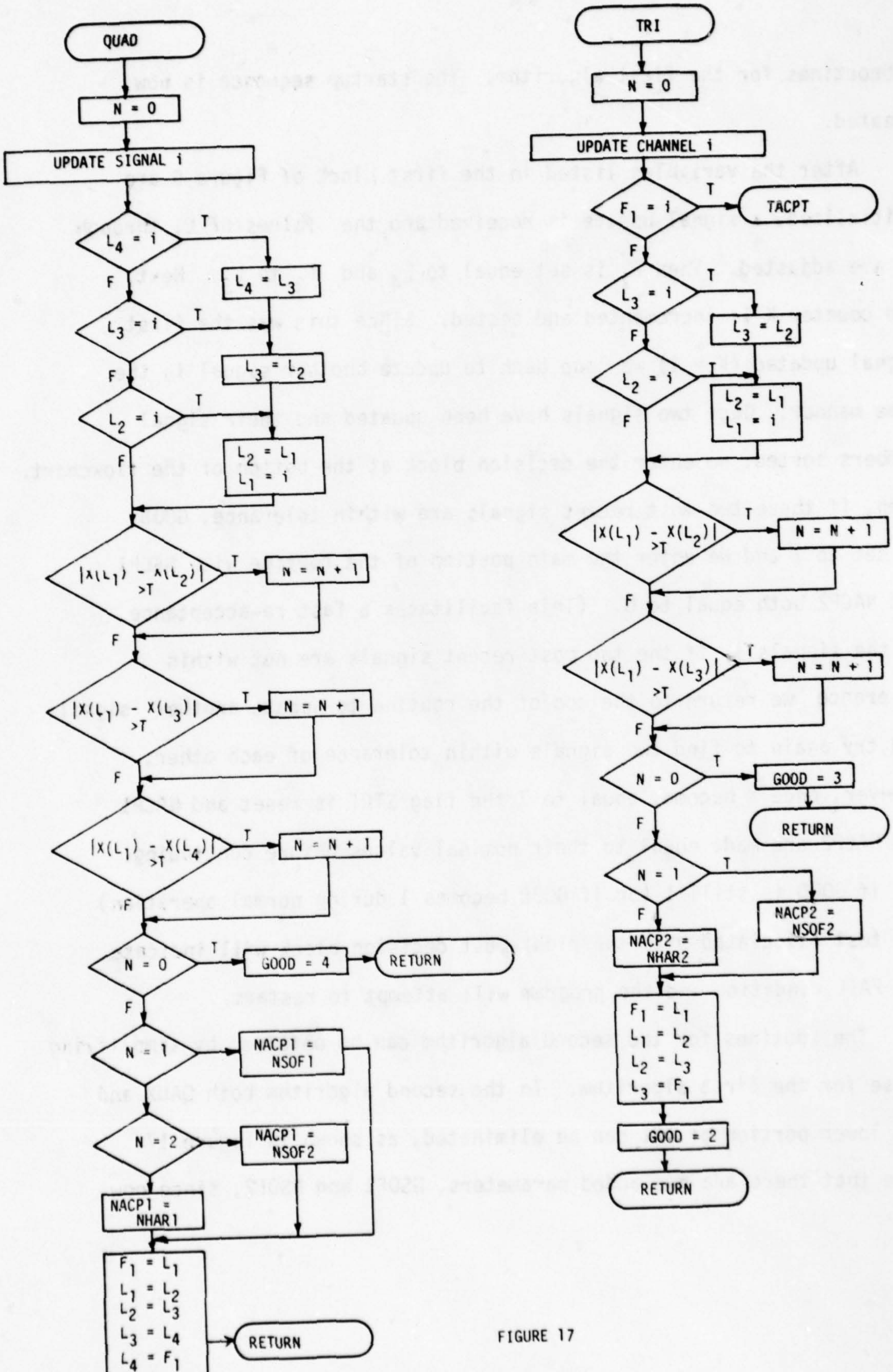


FIGURE 17

SECOND ALGORITHM

there are two levels of soft faults. Otherwise, the required operations are about the same as those for the first algorithm.

The second algorithm has increased sensitivity to soft faults. One way to overcome the sensitivity is to increase the prespecified tolerance, but this may not be desirable. Simulation could be used to resolve this question.

4.0 EVALUATION, COMPARISONS, AND CONCLUSIONS

4.1 Computer Simulation of the Algorithm

The basic algorithm of the previous section was programmed and tested using Monte Carlo simulation. All 4 redundant signals were given a unit-step input plus Gaussian noise of various levels. The Gaussian noise was generated from two random numbers y_1 and y_2 via the transformation

$$\text{noise} = \sigma_{\text{noise}} \cos(2\pi y_1) \qquad \ln(1-y_2)$$

where the probability density function of y_1 and y_2 is

$$p_y(a) = \begin{cases} 1 & 0 \leq a < 1 \\ 0 & \text{otherwise} \end{cases}$$

The results of the simulation are shown in Figures 18 and 19. In Figure 18 the values of NACP1 and NACP2 are 1, so that a faulted signal is re-accepted the first time it comes back within tolerance. In Figure 19 NACP1 and NACP2 are increased to 10.

The Figures display the percentage of times (out of 1000 updates) that QUAD, TRI, and BI were used, as a function of the noise level on the input signals and for a tolerance level of 1. Note that the rolloff slope of QUAD past the noise level of 0.2 is greater for the larger values of NACP1 and NACP2. The program was set to restart when a failure occurs. (See Figure 9.) The first restart occurs for σ_{noise} slightly greater than 0.3, as indicated on the plots. The restart itself begins to fail for $\sigma_{\text{noise}} > 0.5$, but the program was set up to continue to try to restart.

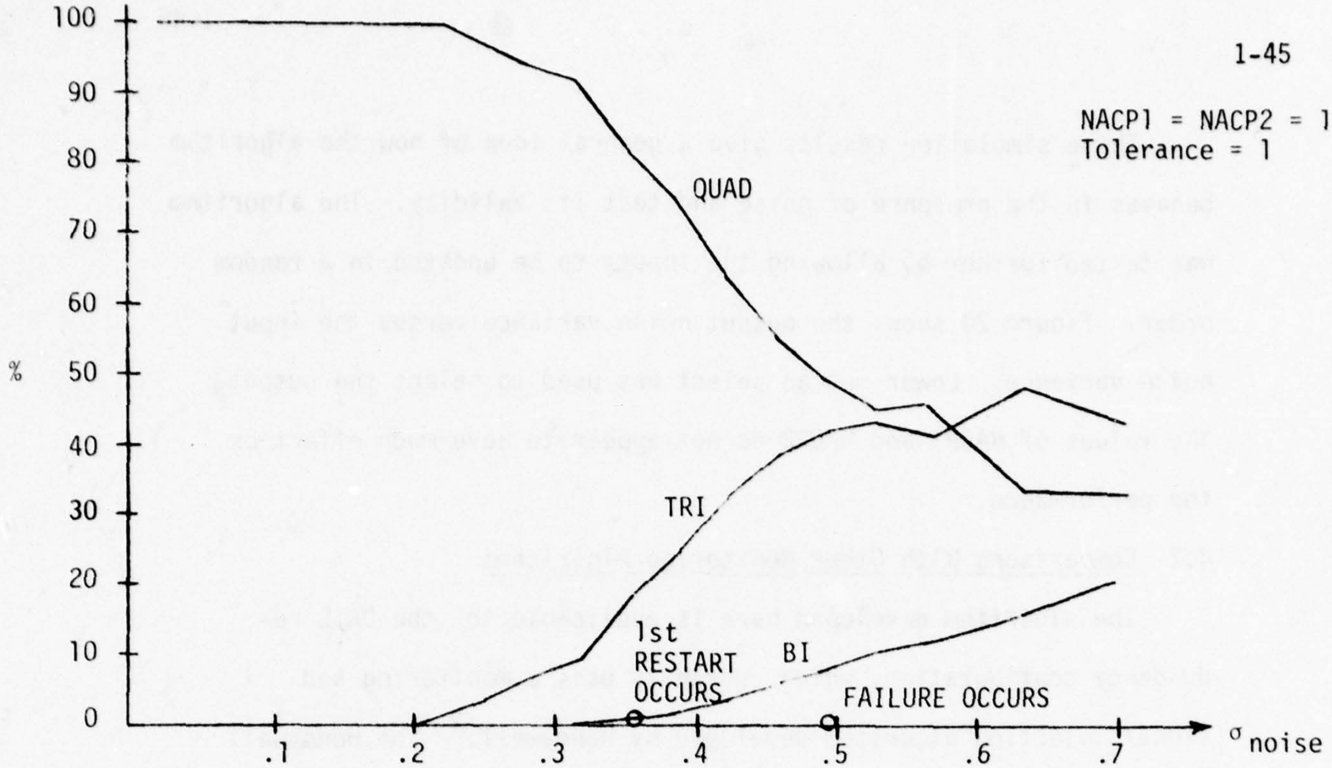


Figure 18 Simulation Results for Low Re-Acceptance Level

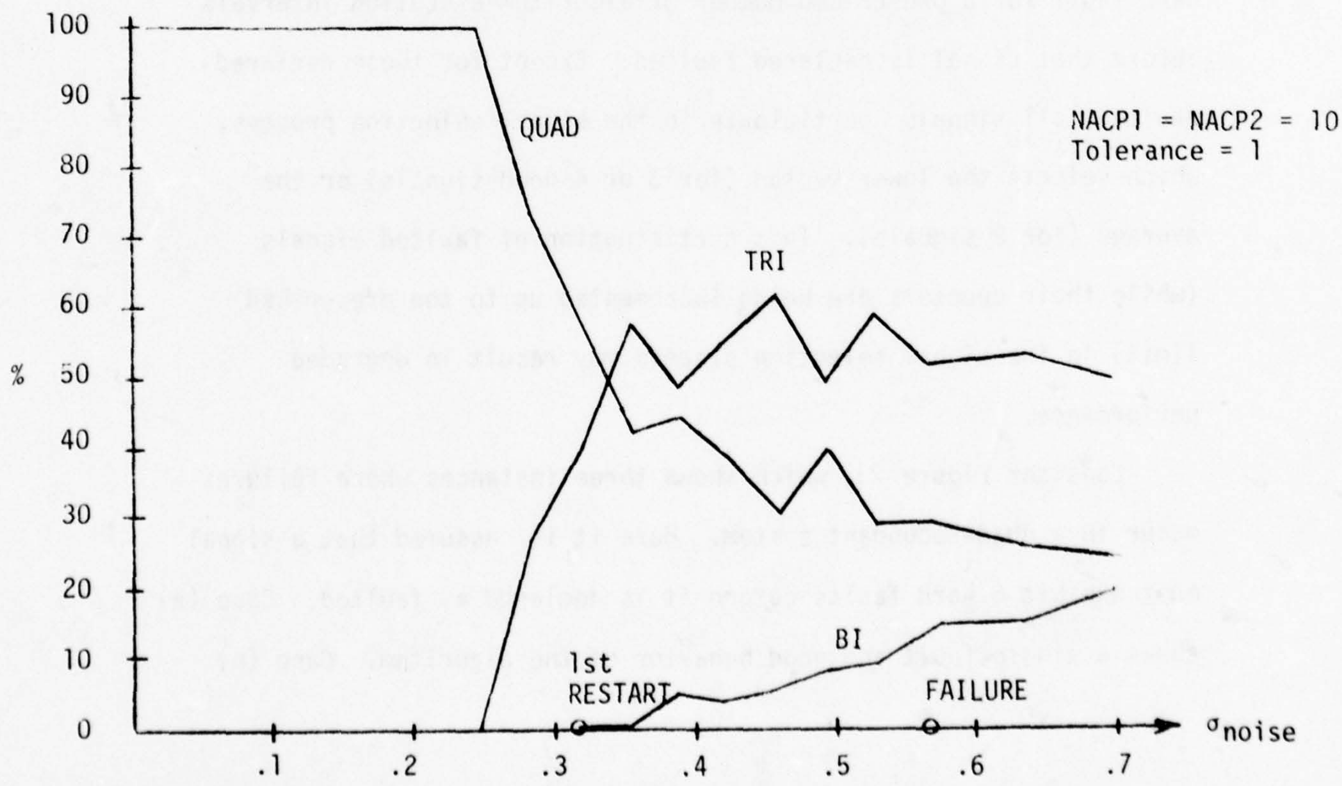


Figure 19 Simulation Results for High Re-Acceptance Level

These simulation results give a general idea of how the algorithm behaves in the presence of noise and test its validity. The algorithm was tested further by allowing the inputs to be updated in a random order. Figure 20 shows the output noise variance versus the input noise variance. Lower-median select was used to select the output. The values of NACP1 and NACP2 do not appear to have much effect on the performance.

4.2 Comparisons With Other Monitoring Algorithms

The algorithm developed here is applicable to the DAIS redundancy configuration, which currently uses a monitoring and signal selection algorithm developed by Honeywell.[3] The Honeywell algorithm performs the same 6 input-input comparison monitoring steps that are a part of the algorithm described in this report. However, the Honeywell algorithm requires that a signal exhibit a hard fault for a prescribed number of algorithm-execution intervals before that signal is declared faulted. Except for those declared faulted, all signals participate in the signal selection process, which selects the lower median (for 3 or 4 good signals) or the average (for 2 signals). This participation of faulted signals (while their counters are being incremented up to the prescribed limit) in the signal selection process may result in degraded performance.

Consider Figure 21, which shows three instances where failures occur in a quad-redundant system. Here it is assumed that a signal must exhibit 6 hard faults before it is declared as faulted. Case (a) shows a single fault and good behavior of the algorithm. Case (b)

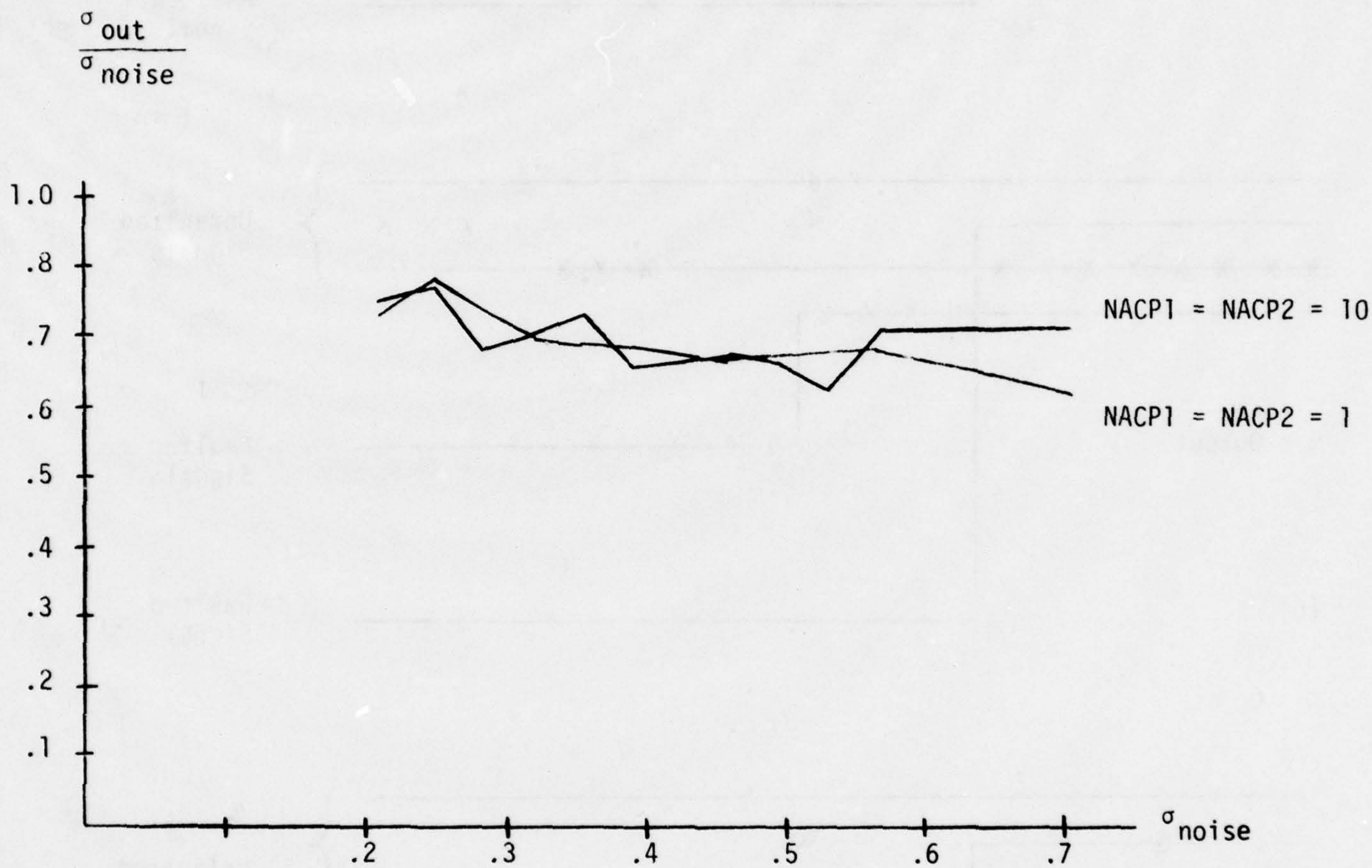


Figure 20 Simulation Results with Inputs Updated in a Random Manner

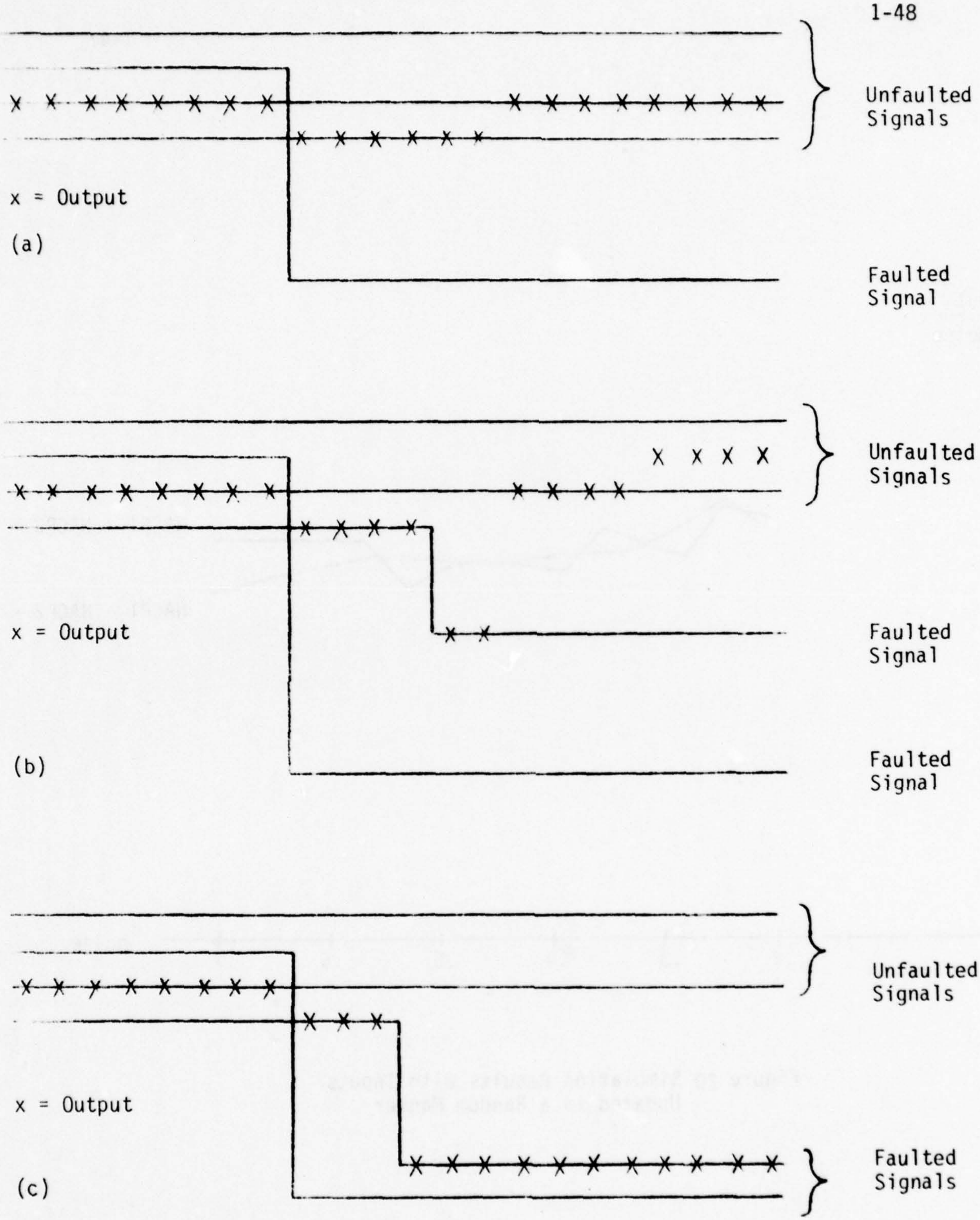


FIGURE 21 THREE CASES SHOWING SIGNAL SELECTION USING THE HONEYWELL ALGORITHM

shows 2 faults--the second occuring before the first was recognized as a fault; here, the algorithm selects a faulted signal as the output but only for a brief time. Case (c) shows a situation where a faulted signal again becomes selected as the output but this time for an unlimited time. It appears that when the fault-counter limit is set high, there is a greater chance that the occurrence of situations like Case (c) will cause the wrong output to be selected.

Note that the Honeywell algorithm detects all single hard faults but does not eliminate the out-of-tolerance signal at the same time that it is detected. The algorithm recognizes all soft faults and a split fault as "glitches" but does not keep track of which signals are involved. The procedure is to count the total number of glitches that occur, regardless of the specific signal pattern. No direct use is made of the glitch count or glitch occurrence by the signal selection algorithm.

The operation of Honeywell algorithm can be expressed in terms of the fault patterns developed in this report. Assume that all 4 signals are participating in the signal-selection process. (Perhaps only because the hard-fault counters have not yet reached their prescribed limits.) Under this assumption Table 7 shows where hard faults and glitches are detected, which signal is selected as the output, and which fault patterns are not dealt with effectively, that is, poor signal selection could possibly take place.

Broen gives the performance of several voters and voter-estimators,[5-7] both of which are designed to filter random variations in individual signals while simultaneously discriminating against a faulted signal.

FAULT LEVEL	PATTERN	REMARK
0		NO FAULTS
1		GLITCH DETECTED
2A		GLITCH DETECTED
2B		GLITCH DETECTED
3A		FAULT DETECTED
3B		GLITCH DETECTED
3C		FAULT DETECTED
4A		FAULT DETECTED
4B	*	GLITCH DETECTED
4C		FAULT DETECTED
5A	*	FAULT DETECTED
5B	*	FAULT DETECTED
5C	*	FAULT DETECTED
6	*	FAULT DETECTED

● = SIGNAL SELECTED
* = PATTERN NOT DEALT WITH EFFECTIVELY

TABLE 7. FAULTS, GLITCHES, AND SIGNAL SELECTION FOR THE HONEYWELL ALGORITHM

The voting techniques use weighting factors chosen so that the effect of one faulted signal out of 3 or 4 measurements will be minimal in comparison with the remaining unfaulted signals. The computations require multiplication and division. Split faults produce outputs that may straddle the split and thus be distant from each of the two groups of signals. This difficulty could possibly be reduced if the quad system were to be reduced to a triplex system by some simple technique such as removing the most positive signal, removing the last failed signal, or removing the last signal with the lowest weight value, etc. Broen's voters could be used as the signal selection device in the algorithm described in this report.

Broen's voter-estimators require a state variable model to calculate the best output.[6,7] Here, the basic idea is to numerically isolate a faulted signal and to apply a least squares estimator to the remaining signals. The total computations involved probably are not feasible for the DAIS hardware but may be feasible where additional hardware is available.

5.0 REFERENCES

1. McDonnel Aircraft Company, "Definition Study For An Advanced Fighter Digital Flight Control System" Air Force Flight Dynamics Laboratory Technical Report AFFDL-TR-75-59, June 1975.

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4. Darcy, V. J. and C. Slivinsky, "Analysis of Inherent Errors in Asynchronous Redundant Digital Flight Control Systems," Technical Report AFFDL-TR-76-16, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, April 1976.

5. Broen, R.B., "New Voters for Redundant Systems," Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control, Vol. 97, Series G, No. 1, March 1975, pp. 41-45.

6. Broen,R.B., "A Fault-Tolerant Estimator for Redundant Systems," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-11, No. 6, November 1975, pp. 1281-1285.

7. Broen, R.B., "Performance of Fault-Tolerant Estimators in a Noisy Environment," AIAA Paper No. 75-1062, AIAA Guidance and Control Conference, Boston, Mass., August 20-22, 1975.

Part II

Models and Software for Closed Loop Operation of Redundant Systems

1.0 INTRODUCTION AND SUMMARY

Reference 1 describes a model for closed-loop flight control systems that have dual-redundant, asynchronous digital controllers. In this model, which we designate the Basic Model, the digital controllers have the same sample rate but there is a fixed time skew, or offset between their respective sample times. This same skewed sampling scheme is used throughout this report to simplify the modeling process.

Three extensions to the model of Reference 1 are developed here, namely:

(1) Multirate Model

(2) Delay Models

(3) Output-Averaging Models

The Multirate Model allows for separate sampling rates for the external inputs (pilot input, wind gusts, etc.) and the digital controllers. The Delay Models allow for computational delays due to the time required for data conversions and control-output computations. The Output-Averaging Model provides for averaging the control outputs produced by each of the redundant controllers, rather than always selecting the output of the same controller, which is the scheme followed in all the other models.

The Basic Model and the three extensions are described in separate sections below. The description of the basic model is in summary form but that of the others is given in detail. Simplified examples are presented for which the calculations can be done by hand.

Application of the model to the study of control systems for realistic aircraft requires the use of the computer. A software package for the Basic Model is described in Reference 1. Corresponding packages for each of the other 3 models is described in separate sections of this report. FORTRAN Program listings and example output are given in the Appendices.

2.0 BASIC MODEL

The model described below is labelled the basic model because the assumptions, techniques, and style of analysis are the basis for the other models described in the remaining sections of this report. The basic model was developed and studied in detail in Reference 1.

In this section the system configuration is described for a closed-loop, dual-redundant system with the voting rule that the same channel (channel 1) is always selected for the output. The system state equations are given without showing the details of their derivation, as these details are available in Reference 1. The covariance analysis developed in Reference 1 is summarized, as are two examples, which are also considered in subsequent sections.

2.1 System Configuration and Dynamic Equations

The closed-loop system configuration for the basic model appears in Figure 1. The system input is assumed to be the continuous vector w_p, which is applied to a continuous time model of the aircraft, the sensor, and the control-actuator dynamics. The system output y_p is sampled by each of two digital controllers; they use the same sample period T, but controller #2 has a skew τ. The inherent error ε is a piecewise-constant function of the controller outputs. The output of Controller #1 serves as the piecewise-constant input to the aircraft, along with the external input vector w_p.

The aircraft, sensor, and actuator dynamics (plot dynamics), as well as any dynamics associated with the external input take the form

$$\dot{x}_p = A_p x_p + B_{1p} u_p + B_{2p} w_p \qquad (2\text{-}1)$$

$$y_p = C_p x_p \qquad (2\text{-}2)$$

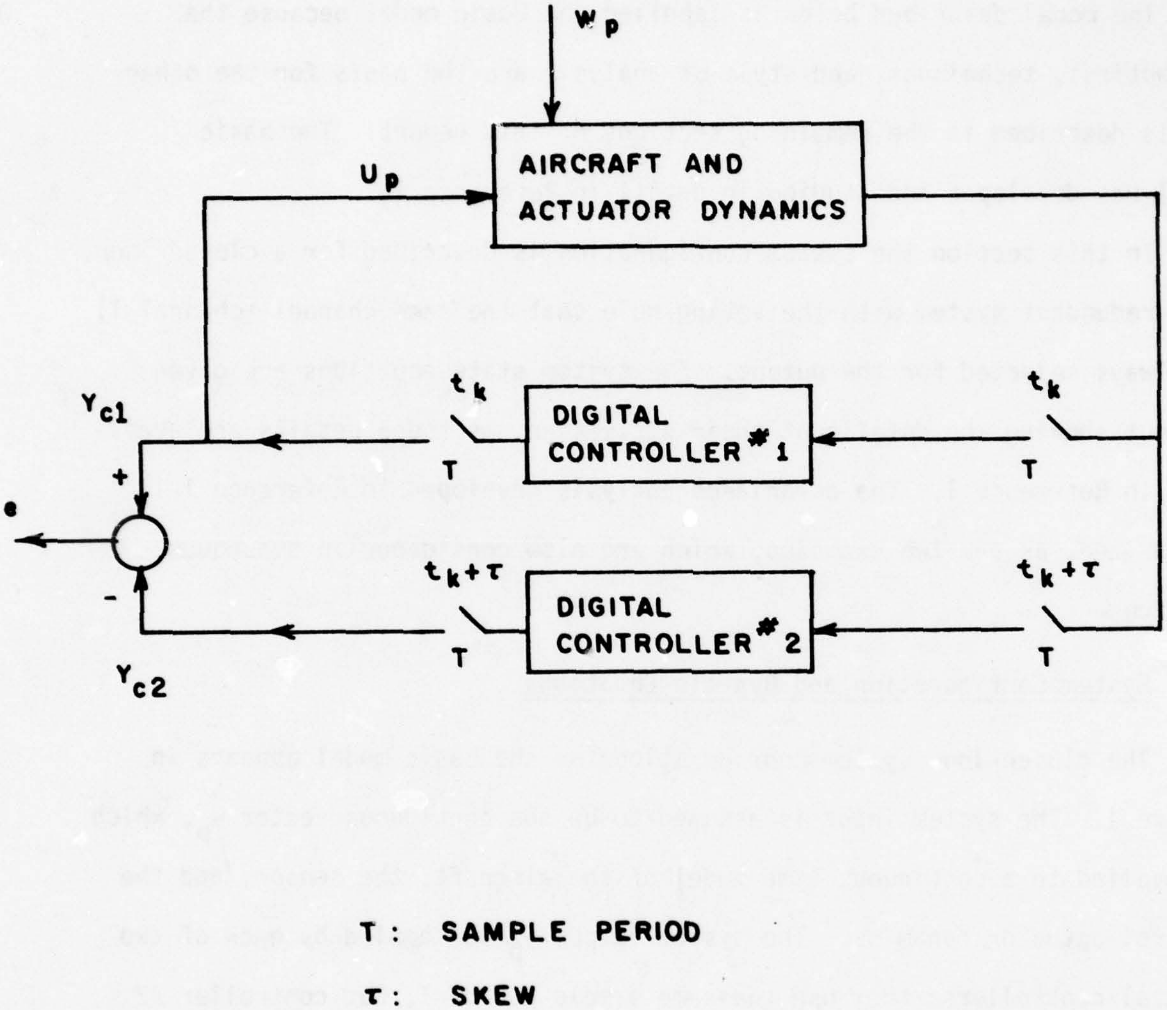


FIGURE 1 BLOCK DIAGRAM FOR THE BASIC MODEL

where

x_p = plant state vector (n_px1)

u_p = plant input vector (n_{up}x1)

w_p = disturbance input vector (n_{wp}x1)

y_p = plant output vector (n_{op}x1)

A_p = plant state matrix (n_pxn_p)

B_{1p} = plant control input matrix (n_pxn_{up})

B_{2p} = plant external input matrix (n_pxn_{wp})

C_p = plant output matrix (n_{op}xn_p)

Controller #1 satisfies the following vector difference equations

$$x_{c1}(t_{k+1}) = F_c x_{c1}(t_k) + G_c u_{c1}(t_k) \tag{2-3}$$

$$y_{c1}(t_k) = H_c x_{c1}(t_k) + E_c u_{c1}(t_k) \tag{2-4}$$

for k = 0, 1,2,... . Controller #2 satisfies

$$x_{c2}(t_{k+1}+\tau) = F_c x_{c2}(t_k+\tau) + G_c u_{c2}(t_k+\tau) \tag{2-5}$$

$$y_{c2}(t_k+\tau) = H_c x_{c2}(t_k+\tau) + E_c u_{c2}(t_k+\tau) \tag{2-6}$$

for k = 0, 1,2,... . Here,

x_{c1} = controller #1 state vector (n_cx1)

u_{c1} = controller #1 control input vector (n_{op}x1)

y_{c1} = controller #1 output vector (n_{up}x1)

x_{c2} = controller #2 state vector (n_cx1)

u_{c2} = controller #2 control input vector (n_{op}x1)

y_{c2} = controller #2 output vector (n_{up}x1)

F_c = controller state matrix (n_cxn_c)

G_c = controller control input matrix ($n_c x n_{op}$)

H_c = controller output matrix (static) ($n_{up} x n_c$)

E_c = controller output matrix (inputs) ($n_{up} x n_{op}$)

The plant equations and the controller equations are related by the following requirements

(1) The control input to the plant is the output of controller #1.

(2) The plant output is the input to both controller #1 and controller #2.

In equation form,

$$u_p(t_k) = y_{c1}(t_k) \tag{2-7}$$

$$u_{c1}(t_k) = y_p(t_k) \tag{2-8}$$

$$u_{c2}(t_k+\tau) = y_p(t_k+\tau) \tag{2-9}$$

The combined equations for closed loop operation take the form

$$x(t_{k+1}) = F(T,\tau)x(t_k) + G(t_k,t_{k+1},\tau,w_p(t))$$

where $x(t_k)$ is a combined state vector consisting of the plant variable x_p and the digital controller variables x_{c1} and x_{c2}, as

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{c1}(t_k) \\ x_{c2}(t_k+\tau) \end{bmatrix}, \quad x(t_{k+1}) = \begin{bmatrix} x_p(t_{k+1}) \\ x_{c1}(t_{k+1}) \\ x_{c2}(t_{k+1}+\tau) \end{bmatrix} \tag{2-10}$$

and

$$F(T,\tau) = \begin{bmatrix} \Phi(t_{k+1},t_k) + \psi(t_{k+1},t_k)E_cC_p & \psi(t_{k+1},t_k)H_c & 0 \\ G_cC_p & F_c & 0 \\ G_cC_p[\Phi(t_k+\tau,t_1) + \psi(t_k+\tau,t_k)E_cC_p] & G_cC_p\psi(t_k+\tau,t_k)H_c & F_c \end{bmatrix} \tag{2-11}$$

$$G(t_k,t_{k+1},\tau,w_p(t)) = \begin{bmatrix} \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},v)B_{2p}w_p(v)dv \\ 0 \\ G_cC_p \int_{t_k}^{t_k+\tau} \Phi(t_k+\tau,v)B_{2p}w_p(v)dv \end{bmatrix} \tag{2-12}$$

where $\Phi(t,v)$ is the state transition matrix and for constant A_p is given by

$$\Phi(t,v) = \exp[A_p(t-v)] \tag{2-13}$$

The controller output equations are

$$y_{c1}(t_k) = H_1x(t_k) \tag{2-14}$$

$$y_{c2}(t_k+\tau) = H_2x(t_k) + E_cC_p\int_{t_k}^{t_k+\tau} \Phi(t_k+\tau,v)B_{2p}w_p(v)dv \tag{2-15}$$

where

$$H_1 = [E_cC_p \quad H_c \quad 0] \tag{2-16}$$

$$H_2 = [E_cC_p[\Phi(t_k+\tau,t_k) + \psi(t_k+\tau,t_k)E_cC_p] \; E_cC_p\psi(t_k+\tau,t_k)H_c] \tag{2-17}$$

The piecewise-constant inherent error $e(t)$ is written as two expressions, $e_A(t)$ and $e_B(t)$, as

$$\begin{aligned} e_A(t) &= y_{c1}(t_k) - y_{c2}(t_k+\tau) \\ &= (H_1-H_2)x(t_k)-E_cC_p\int_{t_k}^{t_k+\tau} \Phi(t_k+\tau,s)B_{2p}w_p(s)ds \end{aligned} \tag{2-18}$$

for $t_k+\tau \leq t < t_{k+1}$, $0 \leq \tau < T$, $k=0,1,\ldots$, and

$$
\begin{aligned}
e_B(t) &= y_{c1}(t_{k+1}) - y_{c2}(t_k+\tau) \\
&= (H_1F-H_2)x(t_k) + \quad H_1\int_{t_k}^{t_{k+1}} \Phi(t_{k+1},s)B_{2p}w_p(s)ds \\
&\qquad H_1G_cC_p \int_{t_k}^{t_k+\tau} \Phi(t_k+\tau,s)B_{2p}w_p(s)ds \\
&\qquad - E_cC_p \int_{t_k}^{t_k+\tau} \Phi(t_k+\tau,s)B_{2p}w_p(s)ds
\end{aligned}
\tag{2-19}
$$

for $t_{k+1} \leq t < t_{k+1} + \tau$, $0 < \tau \leq T$, $k = 0,1,\ldots$.

2.2 Covariance Analysis

The inherent error can be characterized in a statistical manner. The external input w_p is assumed to be a sample function from a Gaussian white noise random process with 0 mean and to be independent of $x(0)$. The covariance matrix of the states is defined as

$$P_x(k) = E[x(t_k)x^T(t_k)] \tag{2-20}$$

where E is the expected-value operator. $P_x(k)$ can be found by solving

$$P_x(k+1) = F(T,\tau)P_x(k)F^T(T,\tau) + V(T,\tau) \tag{2-21}$$

where

$$V(T,\tau) = \begin{bmatrix} V_o(t) & 0 & V_o(\tau)C_p^TG_c^T \\ 0 & 0 & 0 \\ G_cC_pV_o(\tau) & 0 & G_cC_pV_o(\tau)C_p^TG_c^T \end{bmatrix} \tag{2-22}$$

and

$$V_o(t) = \int_o^t \Phi(t,s)B_{2p}W\, B_{2p}^T\, \Phi^T(t,s)ds$$

The matrix W is the input-disturbance covariance matrix defined by

$$E[w_p(t)w_p^T(\tau)] = W\,\delta(t-\tau) \tag{2-23}$$

for all $t \geq t_o$ and $\tau \geq t_o$, where $\delta(t-\tau)$ is the Dirac delta function.

The steady-state covariance, designated P_{xss} is found by solving the equation.

$$P_{xss} = F(T,\tau)P_{xss}F(T,\tau) + V(T,\tau) \tag{2-24}$$

Let $P_{eA}(t)$ and $P_{eB}(t)$ be the covariances of e_A and e_B, respectively; that is, for example,

$$P_{eA}(t) = E[e_A(t)\, e_A^T(t)] \tag{2-25}$$

Then

$$P_{eA}(t) = (H_1-H_2)P_x(k)(H_1-H_2)^T + E_cC_pV_o(\tau)C_p^TE_c^T \tag{2-26}$$

for $t_k+\tau \leq t < t_{k+1},$ $k=0,1,\ldots,$ $0<\tau<T$, and

$$P_{eB}(t) = [H_1F(T,\tau)-H_2]\,P_x(k)[H_1F(T,\tau)-H_2]^T + E_cC_p[V_o(T) - V_o(\tau)]C_p^TE_c^T \tag{2-27}$$

for $t_{k+1} \leq t < t_{k+1}+\tau$, $k=0,1,\ldots,$ $0<\tau\leq T$.

All of the above equations for the basic model are developed in Reference 1.

2.3 Examples

As the first example consider the system of Figure 2. For this system let $W = \sigma_w^2$. Then the analysis reveals that

$$P_{eAss} = K^2\tau\sigma_w^2(1 + \frac{k\tau}{2-TK}) \qquad (2\text{-}28)$$

for $t_k+\tau \leq t < t_{k+1}$, $0 \leq \tau < T$, $k=0,1,\ldots$, and

$$P_{eBss} = K^2(T-\tau)\sigma_w^2(1 + \frac{K(T-\tau)}{2-TK}) \qquad (2\text{-}29)$$

for $t_{k+1} \leq t < t_{k+1} + \tau$, $0 \leq \tau < T$, $K = 0,1,\ldots$.

P_{eAss} and P_{eBss} are plotted in Figure 3 as a function of τ. The diagrams to the right of each plot show the times corresponding to the values of the controller outputs used to calculate e_A and e_B.

As the second example consider the system of Figure 4. Again let $W = \sigma_w^2$. Then the analysis reveals that

$$P_{eAss} = K^2\tau\sigma_w^2 \left[\frac{2-K(T-\tau)(1+KT)}{2-KT(1+KT)}\right] \qquad (2\text{-}30)$$

for $t_k+\tau \leq t < t_{k+1}$, $K=0,1,\ldots$, $0 \leq \tau < T$ and

$$P_{eBss} = K^2(T-\tau)\,\sigma_w^2 \left[\frac{2-K\tau\,(1+KT)}{2-KT(1+KT)}\right] \qquad (2\text{-}31)$$

for $t_{k+1} \leq t < t_{k+1} + \tau$, $K=0,1,\ldots$, $0 \leq \tau < T$.

P_{eAss} and P_{eBss} are plotted in Figure 5 as a function of τ.

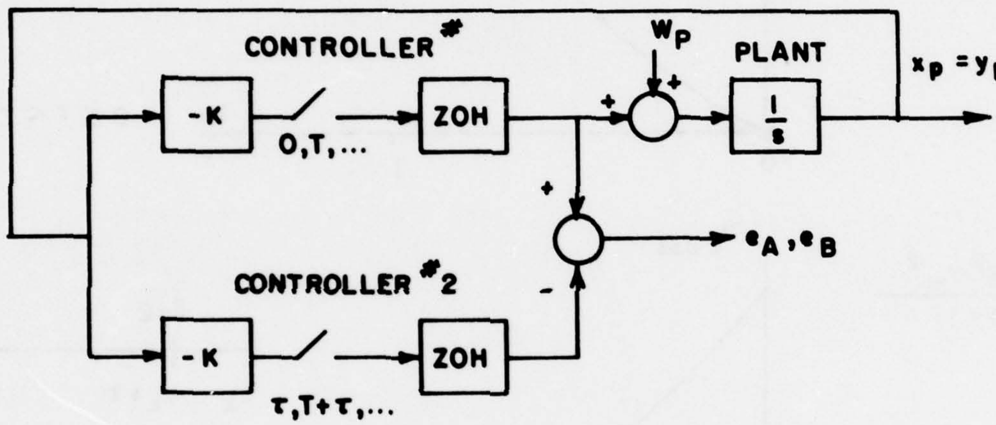


FIGURE 2 EXAMPLE 1 REDUNDANT SYSTEM

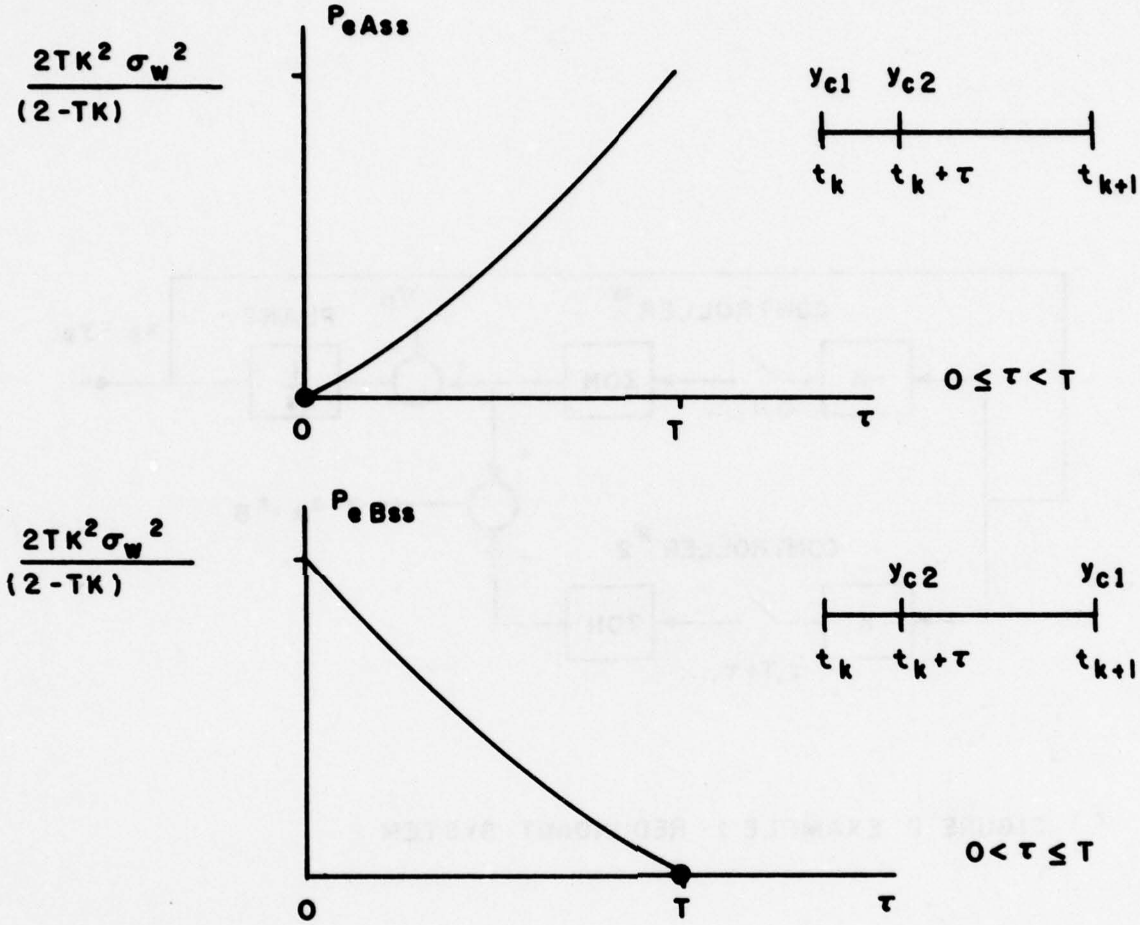


FIGURE 3 P_{eAss} AND P_{eBss} FOR EXAMPLE 1

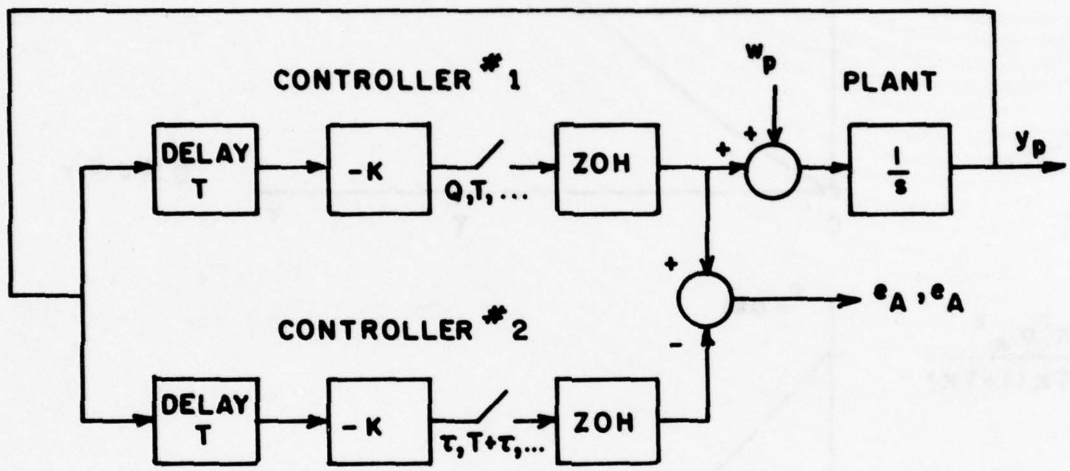


FIGURE 4 EXAMPLE 2 REDUNDENT SYSTEM

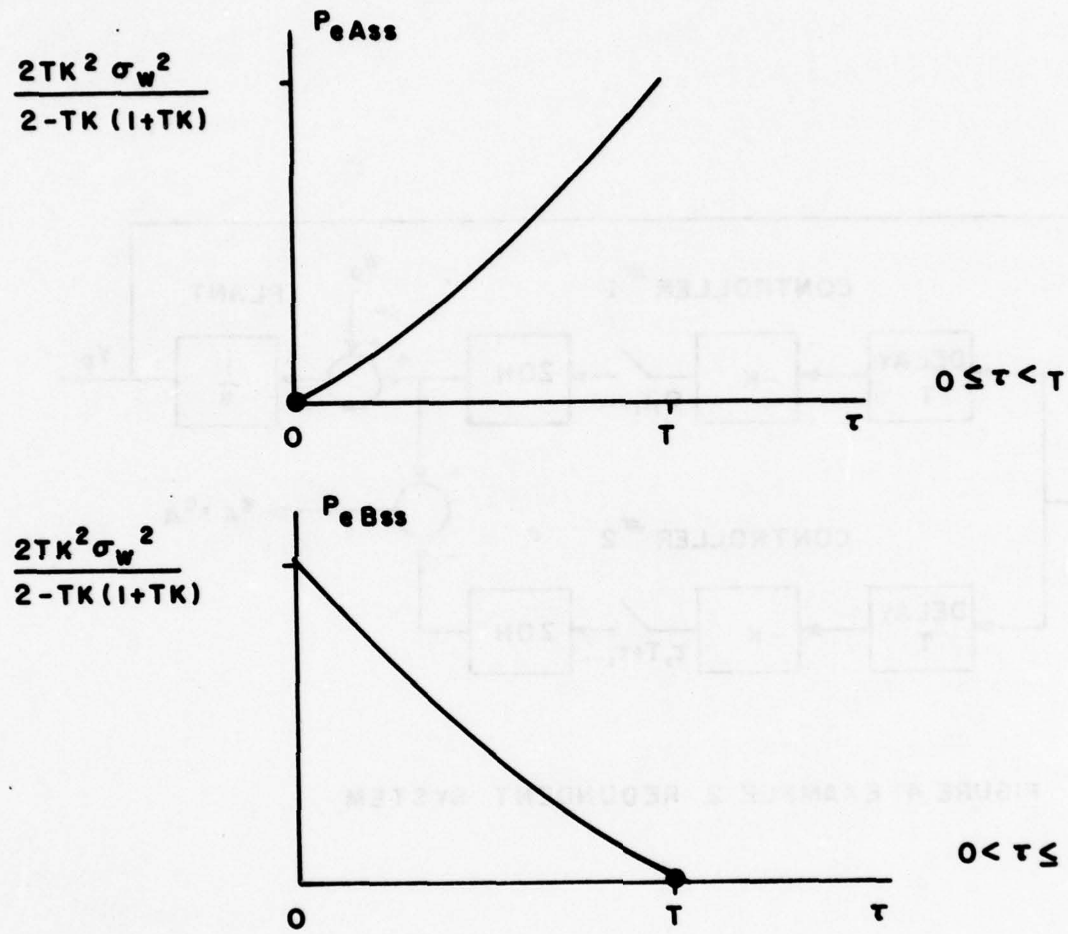


FIGURE 5 P_{eAss} AND P_{eBss} FOR EXAMPLE 2

Note that the results obtained for both examples agree with intuition. For small τ, the variance of e_A is small and that of e_B is large. For large τ the complementary situation holds.

3.0 MULTIRATE MODEL

There are two features that distinguish the Multirate Model from the Basic Model, namely,

(1) The external input is sampled using a sample period T.

(2) The controllers operate at a sample period $\frac{T}{n}$ where n is a positive integer.

In all other respects the components of the Multirate Model are the same as the Basic Model. Thus, in the development below, the same state variables, control variables, matrices, etc., are used as in Section 2.

3.1 System Configuration and Dynamic Equations

The system configuration for the Multirate Model is shown in Figure 6. Note that the external input w_p is now sampled and at a slower rate than the rate at which the control system operates.

For the aircraft, actuator, and sensor dynamics we may write

$$\dot{x}_p = A_p x_p + B_{1p} u_p + B_{2p} w_p \tag{3-1}$$

$$y_p = C_p x_p \tag{3-2}$$

for which the solution is

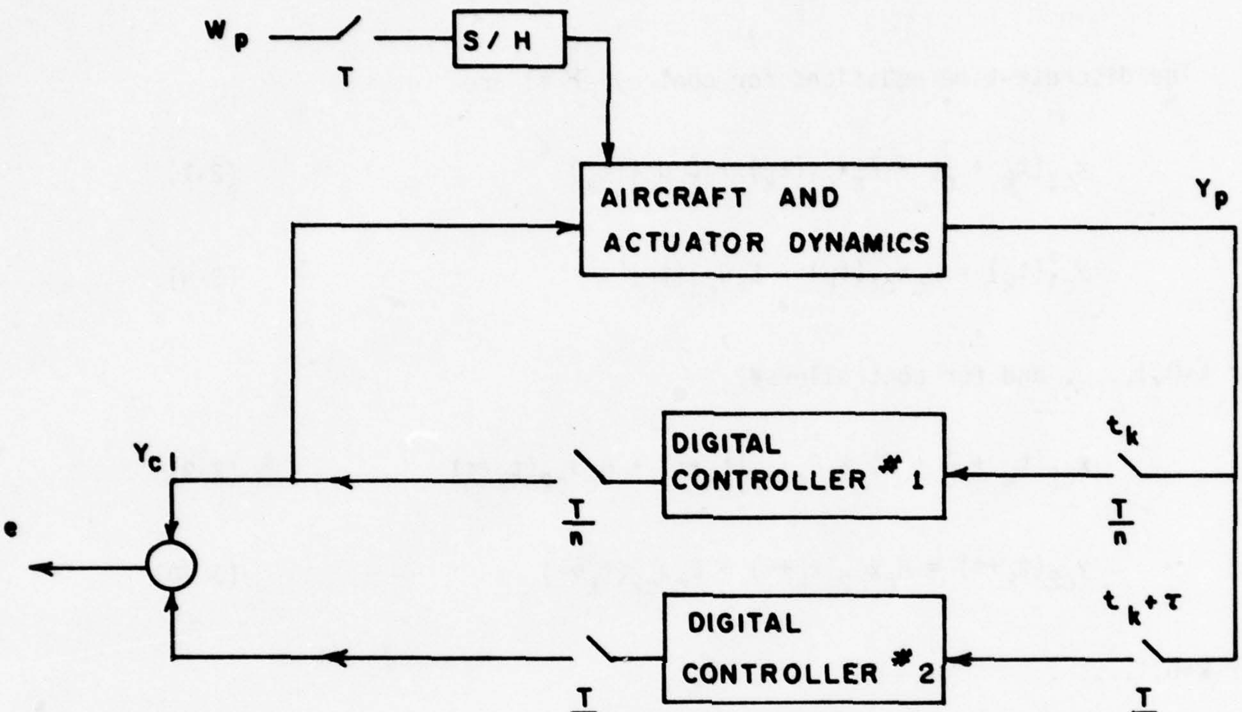
$$x_p(t) = \Phi(t,t_o)x_p(t_o) + \int_{t_o}^{t} \Phi(t,s)B_{1p}u_p(t)ds + \int_{t_o}^{t} \Phi(t,s)B_{2p}w_p(t_o)ds \tag{3-3}$$

Let $t_o=t_k$ and $t = t_k + \frac{T}{n}$. Then (3-3) becomes

$$x_p(t_k + \tfrac{T}{n}) = \Phi(t_k + \tfrac{T}{n},t_k)x_p(t_k) + \psi_1(t_k + \tfrac{T}{n},t_k)u_p(t_k) + \psi_2(t_k + \tfrac{T}{n},t_k)w_p(t_k) \tag{3-4}$$

where

$$\psi_1(t_k + \tfrac{T}{n},t_k) = \int_{t_k}^{t_k+\frac{T}{n}} \Phi(t_k + \tfrac{T}{n},t_k)B_{1p}ds \tag{3-5}$$



S/H : SAMPLE-AND-HOLD

T : PILOT-INPUT SAMPLE PERIOD AND $t_{k+1} - t_k = T$

T/n : DIGITAL-CONTROLLER SAMPLE PERIOD (n A POSITIVE INTEGER)

τ : SKEW

FIGURE 6 BLOCK DIAGRAM FOR THE MULTIRATE MODEL

and
$$\psi_2(t_k + \frac{T}{n}, t_k) = \int_{t_k}^{t_k+\frac{T}{n}} \Phi(t_k + \frac{T}{n}, t_k) B_{2p} ds \quad (3\text{-}6)$$

The discrete-time equations for controller #1 are

$$x_{c1}(t_k + \frac{T}{n}) = F_c x_{c1}(t_k) + C_c u_{c1}(t_k) \quad (3\text{-}7)$$

$$y_{c1}(t_k) = H_c x_{c1}(t_k) + E_c u_{c1}(t_k) \quad (3\text{-}8)$$

for k=0,1,..., and for controller #2,

$$x_{c2}(t_k + \frac{T}{n} + \tau) = F_c x_{c2}(t_k+\tau) + G_c u_{c2}(t_k+\tau) \quad (3\text{-}9)$$

$$y_{c2}(t_k+\tau) = H_c x_{c2}(t_k+\tau) + E_c u_{c2}(t_k+\tau) \quad (3\text{-}10)$$

for k=0,1,... .

The plant (aircraft, actuator, and sensor dynamics) and the controllers are related by the equations

$$u_p(t_k) = y_{c1}(t_k) \quad (3\text{-}11)$$

$$u_{c1}(t_k) = y_p(t_k) = C_p x_p(t_k) \quad (3\text{-}12)$$

$$u_{c2}(t_k+\tau) = y_p(t_k+\tau) = C_p x_p(t_k+\tau) \quad (3\text{-}13)$$

Substituting (3-8) for y_{c1} and (3-12) for u_{c1} in (3-11) yields

$$u_p(t_k) = H_c x_{c1}(t_k) + E_c C_p x_p(t_k) \quad (3\text{-}14)$$

Then substituting (3-14) into (3-4) gives

$$x_p(t_k + \frac{T}{n}) = \Phi(t_k + \frac{T}{n}, t_k)x_p(t_k) + \psi_1(t_k + \frac{T}{n}, t_k)[H_c x_{c1}(t_k) + E_c C_p x_p(t_k)]$$

$$+ \psi_2(t_k + \frac{T}{n}, t_k)w_p(t_k) = [\Phi(\frac{T}{n}) + \psi_1(t_k + \frac{T}{n}, t_k)E_c C_p]x_p(t_k)$$

$$+ \psi_1(t_k + \frac{T}{n}, t_k)H_c x_{c1}(t_k) + \psi_2(t_k + \frac{T}{n}, t_k)w_p(t_k) \tag{3-16}$$

This equation for $x_p(t_k + \frac{T}{n})$ has no terms involving $u_p(t_k)$.

In a similar manner, substituting (3-12) into (3-7) and (3-8) gives

$$x_{c1}(t_k + \frac{T}{n}) = F_c x_{c1}(t_k) + G_c C_p x_p(t_k) \tag{3-17}$$

and

$$y_{c1}(t_k) = H_c x_{c1}(t_k) + E_c C_p x_p(t_k) \tag{3-18}$$

From (3-3) with $t_o = t_k$ and $t = t_k+\tau$, we have

$$x_p(t_k+\tau) = [\Phi(\tau) + \psi_1(t_k+\tau, t_k)E_c C_p]\, x_p(t_k) + \psi_1(t_k+\tau, t_k)H_c x_{c1}(t_k)$$

$$+ \psi_2(t_k+\tau, t_k)w_p(t_k) \tag{3-19}$$

Then substituting (3-13) and (3-19) into (3-9) and (3-10) gives

$$x_{c2}(t_k + \frac{T}{n} + \tau) = F_c x_{c2}(t_k+\tau) + G_c C_p[\Phi(\tau) + \psi_1(t_k+\tau, t_k)E_c C_p]x_p(t_k)$$

$$+ G_c C_p\, \psi_1(t_k+\tau, t_k)H_c x_{c1}(t_k) + G_c C_p \psi_2(t_k+\tau, t_k)w_p(t_k) \tag{3-20}$$

and

$$y_{c2}(t_k+\tau) = H_c x_{c2}(t_k+\tau) + E_c C_p[\Phi(\tau) + \psi_1(t_k+\tau, t_k)E_c C_p]x_p(t_k)$$

$$+ E_c C_p \psi_1(t_k+\tau, t_k)H_c x_{c1}(t_k) + E_c C_p \psi_2(t_k+\tau, t_k)w_p(t_k) \tag{3-21}$$

The state and controller equations can be put in compact form by writing them in terms of a combined state vector. Let

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{c1}(t_k) \\ x_{c2}(t_k+\tau) \end{bmatrix} \quad \text{and} \quad x(t_k+\tfrac{T}{n}) = \begin{bmatrix} x_p(t_k+\tfrac{T}{n}) \\ x_{c1}(t_k+\tfrac{T}{n}) \\ x_{c2}(t_k+\tfrac{T}{n}+\tau) \end{bmatrix}$$

The state equations become

$$x(t_k + \tfrac{T}{n}) = \Delta F\, x(t_k) + \Delta G\, w_p(t_k) \qquad (3\text{-}22)$$

where

$$\Delta F = \begin{bmatrix} \Phi(\tfrac{T}{n}) + \psi_1(\tfrac{T}{n})E_cC_p & \psi_1(\tfrac{T}{n})H_c & 0 \\ G_cC_p & F_c & 0 \\ G_cC_p[\Phi(\tau) + \psi_1(\tau)E_cC_p] & G_cC_p\psi_1(\tau)H_c & F_c \end{bmatrix} \qquad (3\text{-}23)$$

and

$$\Delta G = \begin{bmatrix} \psi_2(\tfrac{T}{n}) \\ 0 \\ G_cC_p\psi_2(\tfrac{T}{n}) \end{bmatrix} \qquad (3\text{-}24)$$

For every transition interval $\{(t_k, t_k + \frac{T}{n}), (t_k + \frac{T}{n}, t_k + \frac{2T}{n}) \ldots, (t_k + \frac{(n-1)T}{n}, t_k+T)\}$ $\psi_1(t_k + \frac{T}{n}, t_k)$, $\psi_1(t_k + \frac{2T}{n}, t_k + \frac{T}{n}), \ldots, \psi_1(t_k+\tau, t_k + \frac{(n-1)T}{n})$ are equal. Then let $t_k=0$ in equation (3-5) we have $\psi_1(t_k + \frac{T}{n}, t_k) = \psi_1(t\ n)$. Similarly $\psi_1(t_k+\tau, t_k)$, $\psi_2(t_k+T/n, t_k)$ and $\psi_2(t_k+\tau, t_k)$ can write in $\psi_1(\tau)$, $\psi_2(T/n)$ and $\psi_2(\tau)$ respectively, and we use $\psi_1(T/n)$, $\psi_1(\tau)$, and $\psi_2(T/n)$ instead of $\psi_1(t_k+T/n, t_k)$, $\psi_1(t_k+\tau, t_k)$ and $\psi_2(t_k+T/n, t_k)$ in (3-23) and (3-24).

As discussed above, and E_c, C_p, F_c, G_c and H_c are constant. Then F and G hold for every transition interval $\{(t_k, t_k + \frac{T}{n}), (t_k + \frac{T}{n}, t_k + \frac{2T}{n}), \ldots, (t_k + \frac{(n-1)T}{n}, t_k + T)\}$. Therefore

$$x(t_k + \frac{2T}{n}) = \Delta F\ x(t_k + \frac{T}{n}) + \Delta G\ w_p(t_k) \tag{3-25}$$

Substituting (3-22) into (3-25) gives

$$x(t_k + \frac{2T}{n}) = \Delta F^2 x(t_k) + (\Delta F + 1)\Delta G\ w_p(t_k) \tag{3-26}$$

Similarly,

$$x(t_k + \frac{3T}{n}) = \Delta F^3 x(t_k) + (\Delta F^2 + \Delta F + 1)\Delta G\ w_p(t_k) \tag{3-27}$$

and so on.

We can write the general equation for these equations by

$$x(t_k + \frac{mT}{n}) = \Delta F^m x(t_k) + \sum_{i=0}^{m-1} (\Delta F)^i \Delta G\ w_p(t_k) \tag{3-28}$$

where $m=1,2,\ldots,n$.

Now let

$$x(t_{k+1}) = \begin{bmatrix} x_p(t_{k+1}) \\ x_{c1}(t_{k+1}) \\ x_{c2}(t_{k+1+\tau}) \end{bmatrix}$$

Then from (3-28), for m=n

$$x(t_{k+1}) = \Delta F^n\ x(t_k) + \sum_{i=0}^{n-1} (\Delta F)^i \Delta G\ w_p(t_k) \tag{3-29}$$

or

$$x(t_{k+1}) = F(T,\tau)\ x(t_k) + G(T,\tau)\ w_p(t_k) \tag{3-30}$$

where

$$F(T,\tau) = (\Delta F)^n \tag{3-31}$$

and

$$G(T,\tau) = \sum_{i=0}^{n-1} (\Delta F)^i \Delta G \tag{3-32}$$

From (3-18) and (3-21) we can write

$$y_{c1}(t_k) = H_1\, x(t_k) \tag{3-33}$$

and

$$y_{c2}(t_k+\tau) = H_2 x(t_k) + E_c C_p \psi_2(\tau) w_p(t_k) \tag{3-34}$$

where

$$H_1 = [E_c C_p \quad H_c \quad 0] \tag{3-35}$$

and

$$H_2 = [E_c C_p(\Phi(\tau) + \psi_1(\tau) E_c C_p) \quad E_c C_p \psi_1(\tau) H_c \quad H_c] \tag{3-36}$$

As same as (3-25). The controller output equations are

$$y_{c1}(t_k+T/n) = H_1 x(t_k+T/n) \tag{3-37}$$

and

$$y_{c2}(t_k+T/n+\tau) = H_2 x(t_k+T/n) + E_c C_p \psi_2(\tau)\, w_p(t_k) \tag{3-38}$$

The general equation of the controller output equations are

$$y_{c1}(t_k + \frac{mT}{n}) = H_1 x(t_k + \frac{mT}{n}) \tag{3-39}$$

and

$$y_{c2}(t_k + \frac{mT}{n} + \tau) = H_2 x(t_k + \frac{mT}{n}) + E_c C_p \psi_2(\tau)\, w_p(t_k) \tag{3-40}$$

where $m=0,1,\ldots,n$.

Substituting (3-28) into (3-39) and (3-40) gives,

$$y_{c1}(t_k + \frac{mT}{n}) = H_1[(\Delta F)^m\, x(t_k) + \sum_{i=0}^{m-1} (\Delta F)^i \Delta G\, w_p(t_k) \tag{3-41}$$

and

$$y_{c2}(t_k + \frac{mT}{n} + \tau) = H_2(\Delta F)^m x(t_k) + [H_2 \sum_{i=0}^{m-1} (\Delta F)^i \Delta G + E_c C_p \psi_2(\tau)]w_p(t_k) \quad (3\text{-}42)$$

where $m=1,2,\ldots,n$.

It is necessary to derive (3-37), (3-38), (3-39), (3-40), (3-41) and (3-42) because in this model the inherent error is defined as

$$e_{Am}(t) = y_{c1}(t_k + \frac{mT}{n}) - Y_{c2}(t_k + \frac{mT}{n} + \tau) \quad (3\text{-}42)$$

for

$$t_k + \frac{mT}{n} + \tau \leq t < t_k + \frac{m+1}{n} T \left\{ \begin{array}{l} k = 0,1,\ldots \\ m=0,1,\ldots,n-1 \text{ and } 0 \leq \tau < \frac{T}{n} \end{array} \right.$$

and

$$e_{Bm}(t) = y_{c1}(t_k + \frac{m+1}{n} T) - y_{c2}(t_k + \frac{mT}{n} + \tau) \quad (3\text{-}43)$$

for

$$t_k + \frac{m+1}{n} T \leq t < t_k + \frac{m+1}{n} T + \tau \left\{ \begin{array}{l} k=0,1,\ldots \\ m=0,1,\ldots,n-1 \text{ and } 0 < \tau \leq \frac{T}{n} \end{array} \right.$$

Figure 7 shows the skewed sampling and inherent errors of the multi-rate model. Channel 1 produces the sampled outputs at times t_k, $t_k + \frac{T}{n}, \ldots, t_k+1$, for $k=0,1,\ldots$ and channel 2 produces the sampled outputs at times $t_k+\tau$, $t_k + \frac{T}{n} + \tau, \ldots, t_k+1+\tau$.

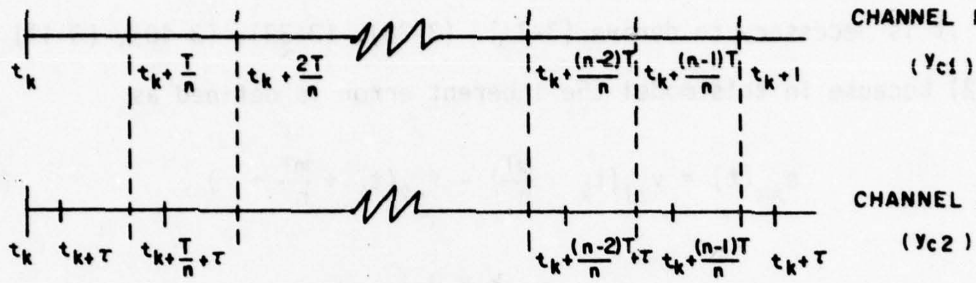
Substituting (3-39) and (3-40) into (3-42) and (3-43) gives

$$e_{Am}(t) = (H_1-H_2)\, x(t_k + \frac{mT}{n}) - E_c C_p \psi_2(\tau) w_p(t_k) \quad (3\text{-}44)$$

and

$$e_{Bm}(t) = (H_1 \Delta F - H_2) x(t_k + \frac{mT}{n}) + [H_1 \Delta G - E_c C_p \psi_2(\tau)]w_p(t_k) \quad (3\text{-}45)$$

where $m=0,1,\ldots,n-1$.



$$e_{Am}(t) = y_{c1}\left(t_k + \frac{mT}{n}\right) - y_{c2}\left(t_k + \frac{mT}{n} + \tau\right)$$

FOR $\quad t_k + \frac{mT}{n} + \tau \le t < t_k + \frac{m+1}{n} T \qquad k = 0, 1,$

$m = 0, 1, \ldots, n-1, \text{and } 0 \le \tau < \frac{T}{n}$

$$e_{Bm}(t) = y_{c1}\left(t_k + \frac{mT}{n} T\right) - y_{c2}\left(t_k + \frac{mT}{n} + \tau\right)$$

FOR $\quad t_k + \frac{m+1}{n} T \le t < t + \frac{m+1}{n} T + \qquad k = 0, 1, \ldots$

$m = 0, 1, \ldots, n-1 \text{ and } 0 < \tau \le \frac{T}{n}$

FIGURE 7 SKEWED SAMPLING AND INHERENT ERRORS

Substituting (3-41) and (3-42) into (3-42) and (3-43) gives $e_{Am}(t)$ and $e_{Bm}(t)$ in terms of $x(t_k)$

$$e_{Am}(t) = (H_1-H_2)\Delta F^m x(t_k) + [(H_1-H_2) \sum_{i=0}^{m-1} (\Delta F)^i \Delta G - E_c C_p \psi_2(\tau)] w_p(t_k) \quad (3\text{-}46)$$

for $t_k + \frac{mT}{n} + T \leq t < t_k + \frac{m+1}{n} T \{ \begin{matrix} k = 0,1,\ldots \\ m = 1,2,\ldots,n-1 \end{matrix}$ and $0 \leq \tau < T/n$

and

$$e_{Bm}(t) = (H_1\Delta F-H_2)(\Delta F)^m x(t_k) + [(H_1\Delta F-H_2) \sum_{i=1}^{m-1} (\Delta F)^i \Delta G + H_1 \Delta G - E_c C_p \psi_2(\tau)] w_p(t_k) \quad (3\text{-}47)$$

for $t_k + \frac{m+1}{n} T \leq t < t_k + \frac{m+1}{n} T + \tau \quad \{ \begin{matrix} k = 0,1,\ldots \\ m = 1,2,\ldots,n-1 \end{matrix}$ and $0 < \tau \leq T/n$

We can write $e_{Am}(t)$ and $e_{Bm}(t)$ when m=0 in terms of $x(t_k)$ by using (3-44) and (3-45)

$$e_{A0}(t) = (H_1-H_2)\, x(t_k) - E_c C_p\, {}_2(\tau) w_p(t_k) \quad (3\text{-}48)$$

for $t_k + T \leq t < t_k + \frac{T}{n}$, $k = 0,1,\ldots$ and $0 \leq T < T/n$

and

$$e_{B0}(t) = (H_1\Delta F-H_2)\, x(t_k) - [H_1 \Delta G - E_c C_p \psi_2(\tau)] w_p(t_k) \quad (3\text{-}49)$$

for $t_k + \frac{T}{n} \leq t < t_k + \frac{T}{n} + \tau$, $k=0,1,\ldots$ and $0 < T \leq T/n$.

From (3-42), we define the inherent error $e_{Am}(t)$ where $m=0,1,\ldots,n-1$ for $\{(t_k+\tau,\ t_k + \frac{T}{n}),\ (t_k + \frac{T}{n} + \tau,\ t_k + \frac{2T}{n}), \ldots, (t_k + \frac{n-1T}{n} + \tau,\ t_k + T)\}$ so let us define E_A; the average error of e_A in period (t_k, t_k+T).

$$E_A = \frac{1}{n} \sum_{m=0}^{n-1} e_{Am} \quad (3\text{-}50)$$

Substituting (3-46) and (3-48) into (3-50) gives

$$E_A = \frac{1}{n} \sum_{m=0}^{n-1} (H_1-H_2)(\Delta F)^m x(t_k) + \frac{1}{n} \sum_{m=1}^{n-1} [(H_1-H_2) \sum_{i=0}^{m-1} (\Delta F)^i \Delta G] w_p(t_k)$$

$$- \frac{1}{n} \sum_{m=0}^{n-1} E_c C_p \psi_2(\tau) w_p(t_k)$$

$$E_A = \frac{H_1 - H_2}{n} \sum_{m=0}^{n-1} (\Delta F)^m x(t_k) + [\frac{H_1 - H_2}{n} (\sum_{m=i}^{n-1} \sum_{i=0}^{m-1} (\Delta F)^i) \Delta B - E_c C_p \psi_2(\tau)] w_p(t_k) \quad (3\text{-}51)$$

Similarly we define E_B; the average error of e_B in period $(t_k, t_k + T + \tau)$ so

$$E_B = \frac{1}{n} \sum_{m=0}^{n-1} e_{Bm} \ . \qquad (3\text{-}52)$$

Substituting (3-47) and (3-49) into (3-52) gives

$$E_B = \frac{1}{n} \sum_{m=0}^{n-1} (H_1 \Delta F - H_2)(\Delta F)^m \ x(t_k) + \frac{1}{n} \sum_{m=1}^{n-1} [(H_1 \Delta F - H_2) \sum_{i=0}^{m-1} (\Delta F)^i \Delta B] w_p(t_k)$$

$$+ \frac{1}{n} \sum_{m=0}^{n-1} [H_1 \Delta G - E_c C_p \psi_2(\tau)] w_p(t_k)$$

$$= \frac{H_1 \Delta F - H_2}{n} \sum_{m=0}^{n-1} (\Delta F)^m \ x(t_k) + [\frac{H_1 \Delta F - H_2}{n} (\sum_{m=i}^{n-1} \sum_{i=0}^{m-1} (\Delta F)^i) \Delta G + H_1 \Delta G$$

$$- E_c C_p \psi_2(\tau)] \ w_p(t_k) \qquad (3\text{-}53)$$

Note: The factor $\sum_{m=1}^{n-1} \sum_{i=0}^{m-1} (\Delta F)^i$ can be expressed in closed form:

$$\sum_{m=1}^{n-1} \sum_{i=0}^{m-1} (\Delta F)^i = \sum_{m=1}^{n-1} \frac{1-(\Delta F)^m}{1- F} = \sum_{m=1}^{n-1} (\frac{1}{1-\Delta F} = \frac{(\Delta F)^m}{1-\Delta F})$$

$$= \frac{n-1}{1-\Delta F} - \sum_{m=0}^{n-1} \frac{(\Delta F)^m}{1-\Delta F} + \frac{1}{1-\Delta F} = \frac{n}{1-\Delta F} - \frac{1-(\Delta F)^n}{(1-\Delta F)^2}$$

3.2 Covariance Analysis

3.2.1 Covariance of the States

Let the input $w_p(t_k)$ be a Gaussian white noise random process with zero mean which is independent of $x(0)$ (Reference 2). Let $\tilde{\{\cdot\}}$ indicate the expected value; then

$$E[w_p(t_k)] = 0 \tag{3-54}$$

$$E[x(t_o)w_p^T(t_k)] = 0 \tag{3-55}$$

and let W_k be the covariance matrix of $w_p(t_k)$. Then

$$E[w_p(t_k)w_p^T(t_k)] = W_k \tag{3-56}$$

Let us define the covariance matrix of the states as

$$P_x(k,m) = E[x(t_k + \frac{mT}{n})\, x^T(t_k + \frac{mT}{n})] \tag{3-57}$$

and

$$P_x(k,m+1) = E[x(t_k + \frac{(m+1)T}{n})x^T(t_k + \frac{(m+1T)}{n})] \tag{3-58}$$

$$= E\{[\Delta Fx(t_k + \frac{mT}{n}) + \Delta Gw_p(t_k)][\Delta Fx(t_k + \frac{mT}{n}) + \Delta Gw_p(t_k)]^T\}$$

$$= \Delta F\, P_x(k,m)(\Delta F)^T + \Delta Gw_k(\Delta G)^T \tag{3-59}$$

where $m=0,1,\ldots,n-1$.

Substituting (3-28) into (3-57) gives $P_x(k,m)$ in term of $P_x(k)$ where $P_x(k)$ is the covariance matrix of $x(t_k)$ and then

$$P_x(k) = E[x(t_k)x^T(t_k)] \tag{3-60}$$

and

$$P_x(k) = P_x(k,0)$$

Then we have

$$P_x(k,m) = (\Delta F)^m P_x(k)(\Delta F^T)^m + \sum_{i=0}^{m-1} (\Delta F)^i \Delta G W_k (\Delta G)^T \sum_{i=0}^{m-1} (\Delta F^T)^i \tag{3-61}$$

Let us define the covariance matrix of the states $P_x(k+1)$ as

$$P_x(k+1) = E[x(t_k+1)\, x^T(t_k+1)] \;. \tag{3-62}$$

Substituting (3-30) into (3-62) gives

$$P_x(k+1) = F(T,\tau)P_x(k)F^T(T,\tau) + G(T,\tau)W_k G^T(T,\tau) \tag{3-63}$$

The steady-state covariance, designated p_{xss} is found by solving the equation

$$P_{xss} = F(T,\tau)\; P_{xss}(T,\tau) + G(T,\tau)\; W_k G^T(T,\tau) \tag{3-64}$$

3.2.2 Covariance of the Errors

The covariances of $e_{Am}(t)$ and $e_{Bm}(t)$ are calculated using the same procedure as in the previous developments. Let P_{eA} be the covariance of e_A then

$$P_{eAm}(t) = E[e_{Am}(t)e_{Am}^T(t)] \tag{3-65}$$

For m=0, substituting (3-48) and (3-60) into (3-65) gives

$$P_{eA0}(t) = (H_1-H_2)P_x(k)(H_1-H_2)^T + E_c C_p \psi_2(\tau) W_k\, {}_2{}^T(\tau) C_p{}^T E_c{}^T \tag{3-66}$$

for $t_k+T \leq t < t_k + \frac{T}{n}$, $k = 0,1,\ldots$

For m>1, substituting (3-46) and (3-60) into (3-65) gives

$$P_{eAm}(t) = (H_1-H_2)(\Delta F)^m P_x(k)[(H_1-H_2)(\Delta F)^m]^T$$
$$+ [(H_1-H_2) \sum_{i=0}^{m-1} (\Delta F)^i \Delta G - E_c C_p \psi_2(\tau)] W_k [(H_1-H_2) \sum_{i=0}^{m-1} (\Delta F)^i \Delta G - E_c C_p \psi_2(\tau)]^T \tag{3-67}$$

for $t_k + \frac{mT}{n} + T \leq t < t_k + \frac{m+1}{n} T$, $\begin{cases} k = 0,1,\ldots \\ m = 1,2,\ldots, n-1 \end{cases}$ and $0 \underline{<} T < T/n$.

Similarly, let P_{eB} be the covariance of e_B then

$$P_{eBm}(t) = E[e_{Bm}(t)\, e_{Bm}^T(t)] \tag{3-68}$$

For m=0, substituting (3-49) and (3-60) into (3-68) gives

$$\begin{aligned} P_{eB0}(t) &= (H_1\Delta F-H_2)P_x(k)(H_1\Delta F-H_2)^T \\ &\quad + [H_1\Delta G-E_cC_p\psi_2(\tau)]\, W_k[H_1\Delta G-E_cC_p\psi_2(\tau)]^T \end{aligned} \tag{3-69}$$

for $t_k + \frac{T}{n} \le t < t_k + \frac{T}{n} + \tau$, $k = 0,1,\ldots$, $0<\tau\le T/n$.

For m 1, substituting (3-47) and (3-60) into (3-68) gives

$$\begin{aligned} P_{eBm}(t) &= [(H_1\Delta F-H_2)(\Delta F)^m]P_x(k)[(H_1\Delta F-H_2)(\Delta F)^m]^T \\ &\quad + [(H_1\Delta F-H_2)\sum_{i=0}^{m-1}(\Delta F)^i\Delta G + H_1\Delta G-E_cC_p\psi_2(\tau)]W_k[(H_1\Delta F-H_2)\sum_{i=0}^{m-1}(\Delta F)^i G \\ &\quad + H_1\Delta G-E_cC_p\psi_2(\tau)]^T \end{aligned} \tag{3-70}$$

for $t_k + \frac{m+1}{n} T \le t < t_k + \frac{m+1}{n} T + \left\{ \begin{array}{l} k=0,1,\ldots \\ m=1,2,\ldots,n-1 \text{ and } 0<\tau\le T/n. \end{array} \right.$

From Figure 6, if n=1 then this model is almost the same as the basic model except that w_p in this model is sampled and zero-order-hold whereas w_p in the basic model is continuous. With n=1, there are only one value of m that is zero. Therefore, if we let n=1 then the covariance errors of this model should be close to or behave the same as the covariance errors of the basic model. In the next section (3.3) we will apply the data from the first example in section 2 (the basic model) with the equations of this model and show that if n=1 then the covariance errors of this model behave the same as the covariance errors of the first example of the basic model as expected.

3.3 Example

By using the data from the first example in Section 2, A_p=0, B_{1p}=1, B_{2p}=1, C_p=1, F_c=0, G_c=0, H_c=0 and E_c=-k. For the zero-order hold of

w_p; $W_k = \frac{\sigma_w^2 n}{T}$ and $t_k + 1 - t_k = T$. Using (3-23) and (3-24) the matrix F and G are calculated to be

$$F = \begin{bmatrix} 1 - \frac{kT}{n} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3-71}$$

and

$$G = \begin{bmatrix} \frac{T}{n} \\ 0 \\ 0 \end{bmatrix} \tag{3-72}$$

Substituting (3-71) and (3-72) into (3-31) and (3-32), the matrix F and G are calculated to be

$$F = \begin{bmatrix} (1 - \frac{kT}{n})^n & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3-73}$$

and

$$G = \begin{bmatrix} \frac{T}{n} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{T}{n}(1 - \frac{kT}{n}) \\ 0 \\ 0 \end{bmatrix} + \dots + \begin{bmatrix} \frac{T}{n}(1 - \frac{kT}{n})^n \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{T}{n}[1 + (1 - \frac{kT}{n}) + (1 - \frac{kT}{n})^2 + \dots + (1 - \frac{kT}{n})^n \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{T}{n}\left[\frac{1-P^n}{1-P}\right] \\ 0 \\ 0 \end{bmatrix} \tag{3-74}$$

where

$$P = 1 - \frac{kT}{n} \tag{3-75}$$

The steady-state covariance of the states is found by solving (3-64)

$$P_{xss} = \frac{\sigma_w^2 T(1-p^n)^2}{n(1-p^{2n})(1-p)^2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3-76}$$

P_{eAmss} and P_{eBmss} can be calculated from (3-66), (3-65), (3-67) and (3-70). From (3-35) and (3-36) we get

$$H_1 = [-k \quad 0 \quad 0] \tag{3-77}$$

and

$$H_2 = [-k(1-k\tau) \quad 0 \quad 0] \tag{3-78}$$

Therefore, we get

$$P_{eAOss} = \frac{k^4\tau^2\sigma_w^2 T\,[1-P^n]^2}{n[1-P^{2n}][1-P]^2} + \frac{k^2\tau^2\sigma_w^2 n}{T} \tag{3-79}$$

$$P_{eBOss} = \frac{k^4\sigma_w^2 T(1-P^n)^2}{n(1-P^{2n})(1-P)^2}\left(\frac{T}{n} - \tau\right)^2 + \frac{k^2\sigma_w^2 n}{T}\left(\frac{T}{n} - \tau\right)^2 \tag{3-80}$$

$$P_{eAmss} = \frac{k^4\tau^2\sigma_w^2 T P^{2m}\,(1-P^n)^2}{n(1-P^{2n})\,(1-P)^2} + \frac{k^2\sigma_w^2\tau^2 n}{T}\left[1 + \frac{k^2T^2(1-P^m)^2}{n^2(1-P)^2} - \frac{2kT(1-P^m)}{n(1-P)}\right] \tag{3-81}$$

where m=1,2,...,n-1. And

$$P_{eBmss} = k^4\left(\frac{T}{n} - \tau\right)^2 \frac{P^{2m}\sigma_w^2 T(1-P^n)^2}{n(1-P^{2n})(1-P)^2} + \frac{k^2\sigma_w^2 n}{T}\left(\frac{T}{n} - \tau\right)^2 \left[\frac{1+k^2\tau^2(1-P^m)}{n^2(1-P)} - \frac{2kT(1-P^m)}{n(1-P)}\right] \tag{3-82}$$

where m=1,2,...,n-1.

As discussed in the previous section, let n=1. Then (3-79) and (3-80) becomes

$$P_{eA0ss} = k^2\tau\sigma_w^{\ 2}\left[\frac{k\tau}{2-kT} + \frac{\tau}{T}\right] \tag{3-83}$$

and

$$P_{eB0ss} = k^2(T-\tau)\,\sigma_w^{\ 2}\left[\frac{k(T-\tau)}{2-kT} + \frac{(T-\tau)}{T}\right] \tag{3-84}$$

P_{eA0ss} and P_{eAss} (equation (2-28), first example in Section 2) are plotted in the same graph in Figure 8 as a function of τ. P_{eB0ss} and P_{eBss} (equation (2-29), first example in Section 2) are plotted in the same graph in Figure 9 as a function of τ. From the graphs of Figure 8 and Figure 9, P_{eA0ss} and P_{eB0ss} behave the same as P_{eAss} and P_{eBss} as expected. P_{eA0ss} and P_{eB0ss} are also equal to P_{eAss} and P_{eBss} at $\tau=0$ and $\tau=T$, but they are less than P_{eAss} and P_{eBss} for $0<\tau<T$.

In this model, we assume to use the same value of the covariance matrix of $w_p(t_k)$ for every interval of time $\{(t_k+1,t_k),\ (t_k+1,t_k+\tau),\ (t_k+\tau,t_k)\}$. It isn't true because for n=1 $E[w_p(t_k)\ w_p^{\ T}(t_k)] = \frac{W}{T}$ (w is the input disturbance covariance matrix) is derived from the noise in period $T(t_k+1,t_k)$. That means, we can use this value $\frac{W}{T}$ in

$$E\left[\int_{t_1}^{t_2} \Phi(t_2,\sigma)\ B_{1p}w_p(t_k)d\sigma \int_{t_1}^{t_2} w_p^{\ T}(t_k)B_{1p}^T\Phi^T(t_2,s)ds\right]$$

if the limit of this integration t_1 and t_2 are t_k and t_k+1. But in the equations of this model we use $\frac{W}{T}$ for every interval of time $\{(t_k+1,t_k),$ $(t_k+1,t_k+\tau),\ (t_k+\tau,t_k)\}$ which makes the difference between P_{eAss} and P_{eA0ss}, P_{eB0ss} and P_{eBss}. The reason for these are as follows;

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INHERENT ERRORS IN ASYNCHRONOUS DIGITAL FLIGHT CONTROLS.(U)

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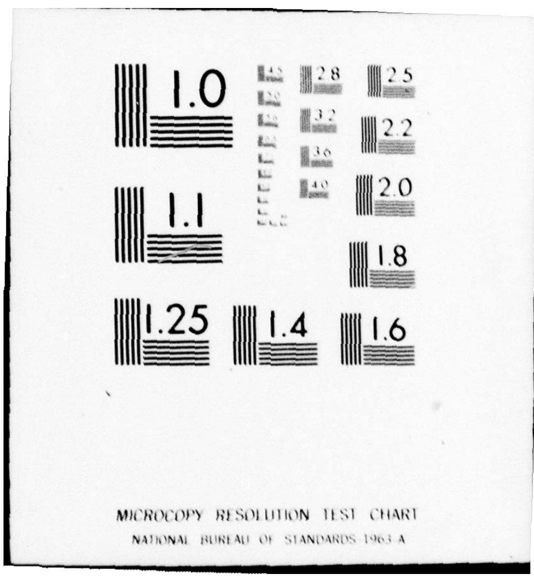
1.0
2.8
2.5
3.2
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MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

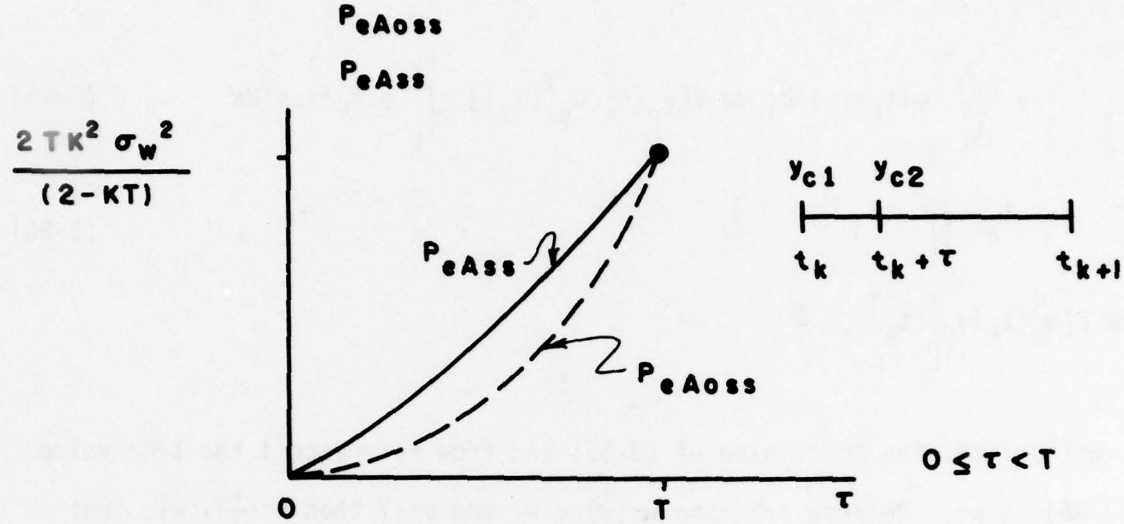


FIGURE 8 P_{eAoss} AND P_{eAss}

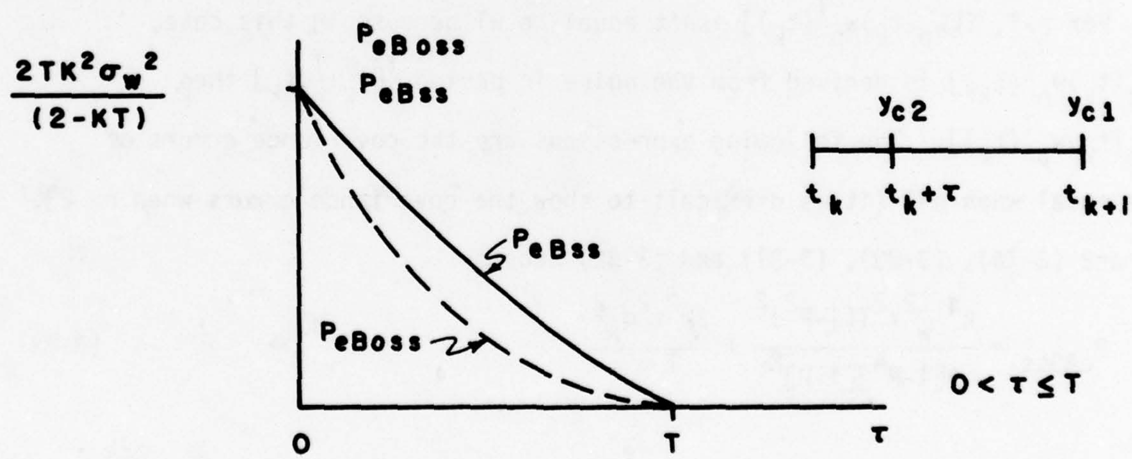


FIGURE 9 P_{eBoss} AND P_{eBss}

With data from this example.

$$E\int_{t_k}^{t_k+\tau} \Phi(t_k+\tau,\sigma)B_{1p}w_p(t_k)d\sigma \int_{t_k}^{t_k+T} w_p^T(t_k)B_{1p}^T \Phi(t_k+\tau,s)ds]$$

$$= \int_{t_k}^{t_k+\tau} \Phi(t_k+\tau,\sigma)B_{1p}d\sigma\ E[w_p(t_k)w_p^T(t_k)] \int_{t_k}^{t_k} \Phi(t_k+\tau,s)ds \qquad (3\text{-}85)$$

$$= w\tau(\tfrac{\tau}{T}) \qquad (3\text{-}86)$$

where $E[w_p(t_k)w_p^T(t_k^T) = \frac{w}{T}$

$w\tau(\frac{\tau}{T})$ isn't the true value of (3-85) and from reference 1 the true value of (3-85) is $w\tau$. Because $\tau<T$ then $w\tau(\frac{\tau}{T}) < w\tau$ and $\Phi\tau\to T$ then $w\tau(\frac{\tau}{T})\to wT$; that means at $\tau=0$ and $\tau=T$ of (3-86) is the true value of (3-85). Therefore we can say that the covariance errors of this model are the approximate values and they are less than the true value which they should be.

For $n>1$, $E[w_p(t_k)w_p^T(t_k)]$ isn't equal to wT because in this case, $E[w_p(t_k)w_p^T(t_k)]$ is derived from the noise in period $\frac{T}{n}[t_k+\frac{T}{n},t_k]$ then $E[w_p(t_k)w_p^T(t_k)]$. The following expressions are the covariance errors of this model when n=2 (it is difficult to show the covariance errors when n> 2]. For n=2 (3-74), (3-80), (3-81) and (3-82) become

$$P_{eAOss} = \frac{k^4\ {}_w{}^2\tau^2T(1-P^2)^2}{2[1-P^4][1-P]^2} + \frac{2k^2\tau^2\sigma_w^{\ 2}}{T} \qquad (3\text{-}84)$$

$$P_{eBOss} = \frac{k^4\sigma_w^{\ 2}T(1-P^2)^2}{2[1-P^4][1-P]^2}\,(\tfrac{T}{2}-\tau)^2 + \frac{2k^2\sigma_w^{\ 2}}{T}\,[\tfrac{T}{2}-\tau]^2 \qquad (3\text{-}85)$$

$$P_{eA1ss} = \frac{k^4\sigma_w^2 P^2\tau^2 T(1-P^2)^2}{2(1-P^4)(1-P)^2} + \frac{2k^2\sigma_w^2\tau^2}{T}[1 + \frac{k^2T^2}{4} - kT] \quad (3\text{-}86)$$

and

$$P_{eB1ss} = \frac{k^4\sigma_w^2 P^2 T(1-P^2)^2}{2(1-P^2)(1-P)^2}(\frac{T}{2} - \tau)^2 + \frac{2k^2\sigma_w^2}{T}(\frac{T}{2} - \tau)^2[1 + \frac{k^2T^2}{4} - kT] \quad (3\text{-}87)$$

P_{wA0ss} and P_{eB0ss} are plotted in Figure 10 and P_{eA1ss} and P_{eB1ss} are plotted in Figure 11. All of them are plotted as a function of τ. The diagram to the right of each plot shows the times corresponding to the values of the controller outputs used to calculate e_A and e_B.

The result obtained for this example agree with intuition. For small τ, the variance of e_A is small and that of e_B is large. For large τ, the complementary situation holds.

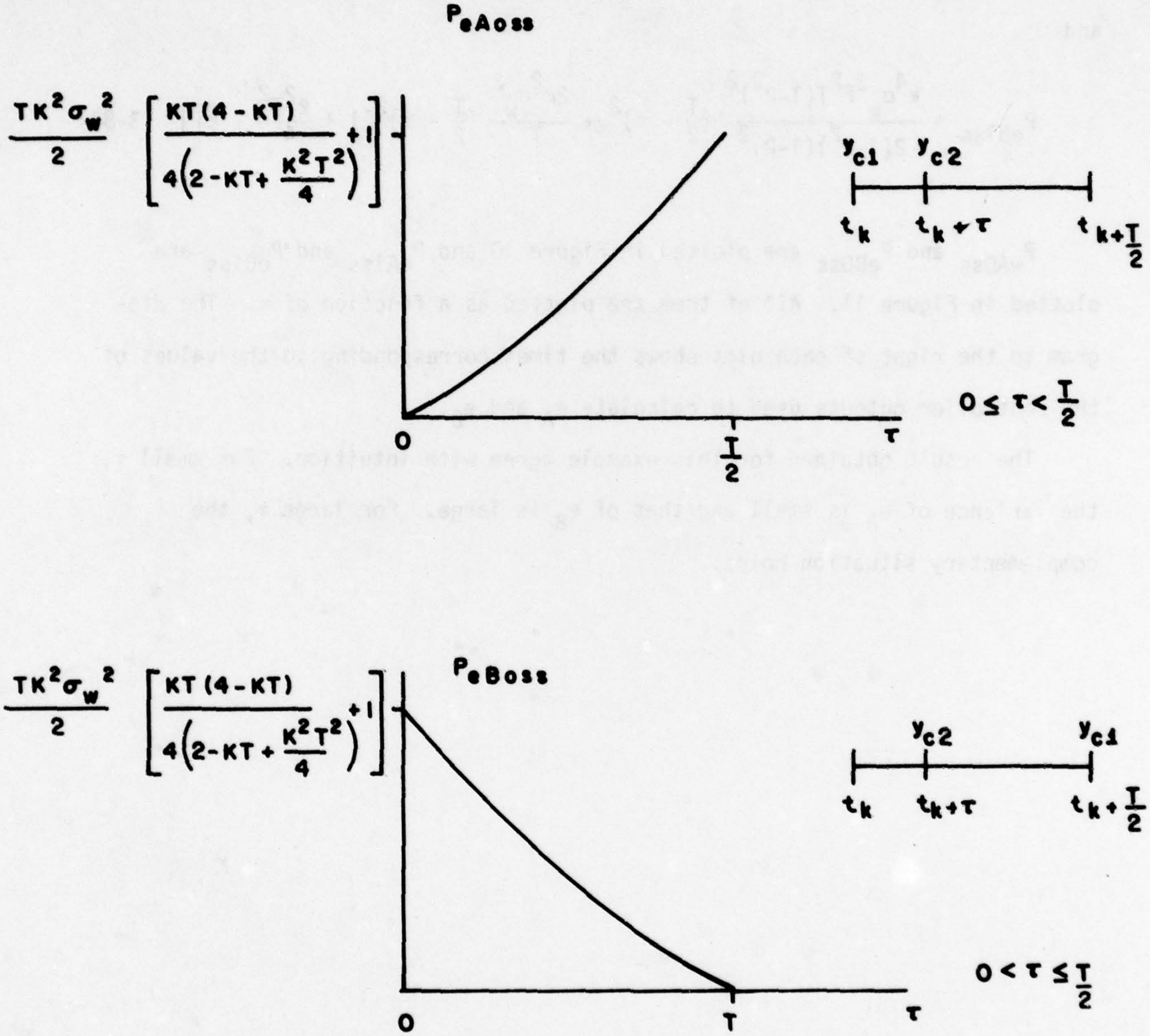


FIGURE 10 P_{eAoss} AND P_{eBoss} $(n=2)$

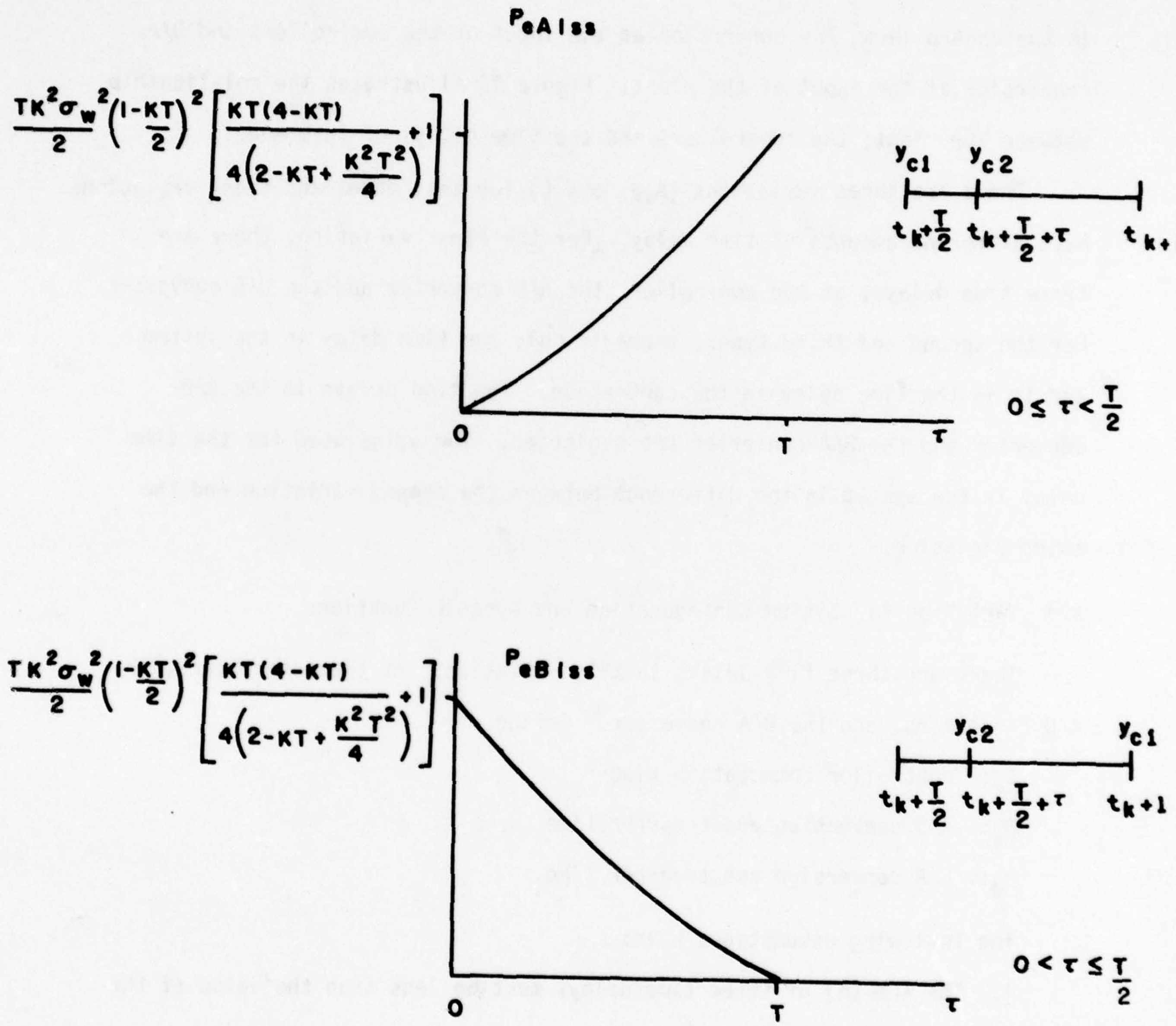


FIGURE 11 P_{eAIss} AND P_{eBIss} (n=2)

4.0 THE DELAY MODEL

The Delay Model is close to the basic model, except there are time delays in the controllers, A/D conversion at the input of the controllers and D/A conversion at the input of the plant. Figure 12 illustrates the relationship between the plant; the controllers and the time delays of this model.

There are three variations (A,B, and C) for this model and these variations have different amounts of time delay. For the first variation, there are three time delays; at the controller, the A/D converter and the D/A converter. For the second and third types, there is only one time delay in the system and it is the time delay in the controller. The time delays in the A/D converter and the D/A converter are neglected. The value used for the time delay in the system is the difference between the second variation and the third variation.

4.1 Variation A: *System Configuration and Dynamic Equations*

There are three time delays in this variation. At the controllers, the A/D convertors, and the D/A converter. Define

δ_c: controller computation time

δ_a: A/D conversion and transfer time

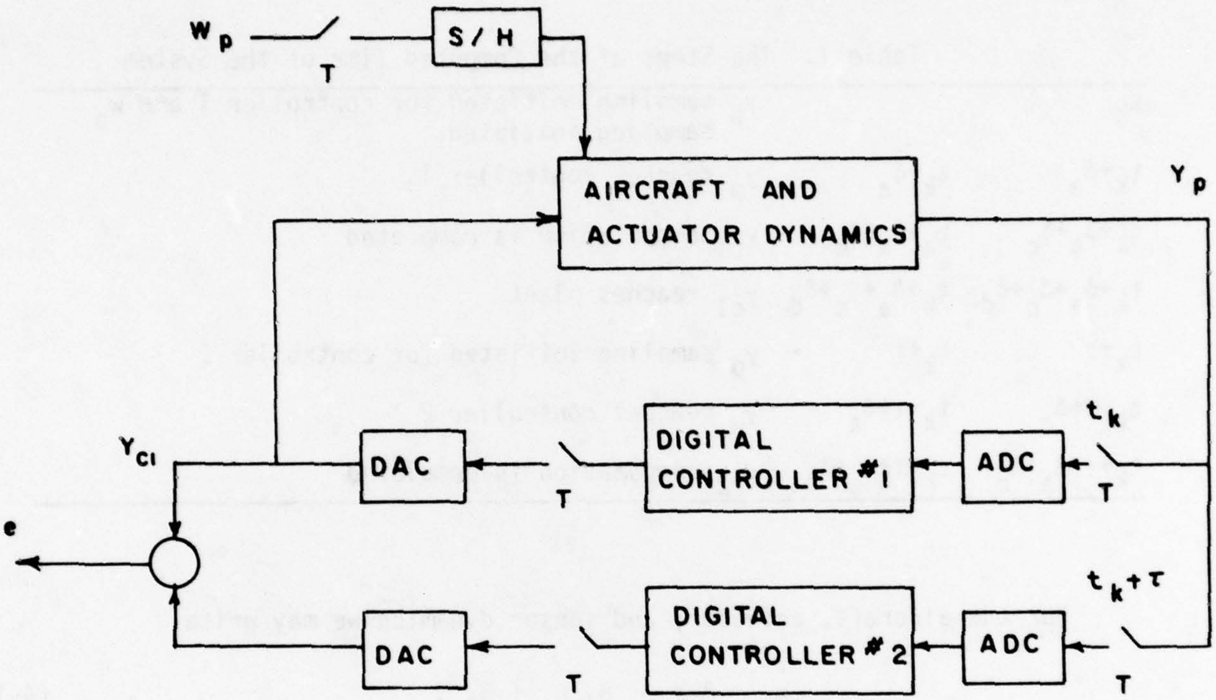
δ_d: D/A conversion and transfer time.

The following assumptions hold:

1. The sum (Δ) of three time delays must be less than the value of the skew time. That is $\Delta = \delta_a+\delta_c+\delta_d$ and $0<\Delta<\tau$.

2. The computation times for the output of the controllers must be complete in the period T.

3. From 2, the value of the skew time must satisfy $\Delta<\tau<T-\Delta$.



ADC : ANALOG-TO-DIGITAL CONVERTER

DAC : DIGITAL-TO-ANALOG CONVERTER

S/H : SAMPLE-AND-HOLD

T : PILOT-INPUT SAMPLE PERIOD AND $t_k+1-t_k=T$

τ : SKEW

FIGURE 12 BLOCK DIAGRAM FOR THE DELAY MODEL

Figure 13 illustrates the time diagram of the state variables and the output of the controllers of the system. Table 1 explains the key events involved in the computations.

Table 1. The Steps of the Computed Time of the System

t_k		y_p sampling initiated for controller 1 and w_p sampling initiated.
$t_k+\delta_a$	$t_k+\delta_a$	y_p reaches controller 1.
$t_k+\delta_a+\delta_c$	$t_k+\delta_a+\delta_c$	y_{c1} computation is completed
$t_k+\delta_a+\delta_c+\delta_d$	$t_k+\delta_a+\delta_c+\delta_d$	y_{c1} reaches plant
$t_k+\tau$	$t_k+\tau$	y_p sampling initiated for controller 2
$t_k+\tau+\delta_c$	$t_k+\tau+\delta_a$	y_p reaches controller 2
$t_k+\tau+\delta_a+\delta_c$	$t_k+\tau+\delta_a+\delta_c$	y_{c2} computation is completed

For the aircraft, actuator, and sensor dynamics we may write

$$\dot{x}_p = A_p x_p + B_{1p} u_p + B_{2p} w_p \tag{4-1}$$

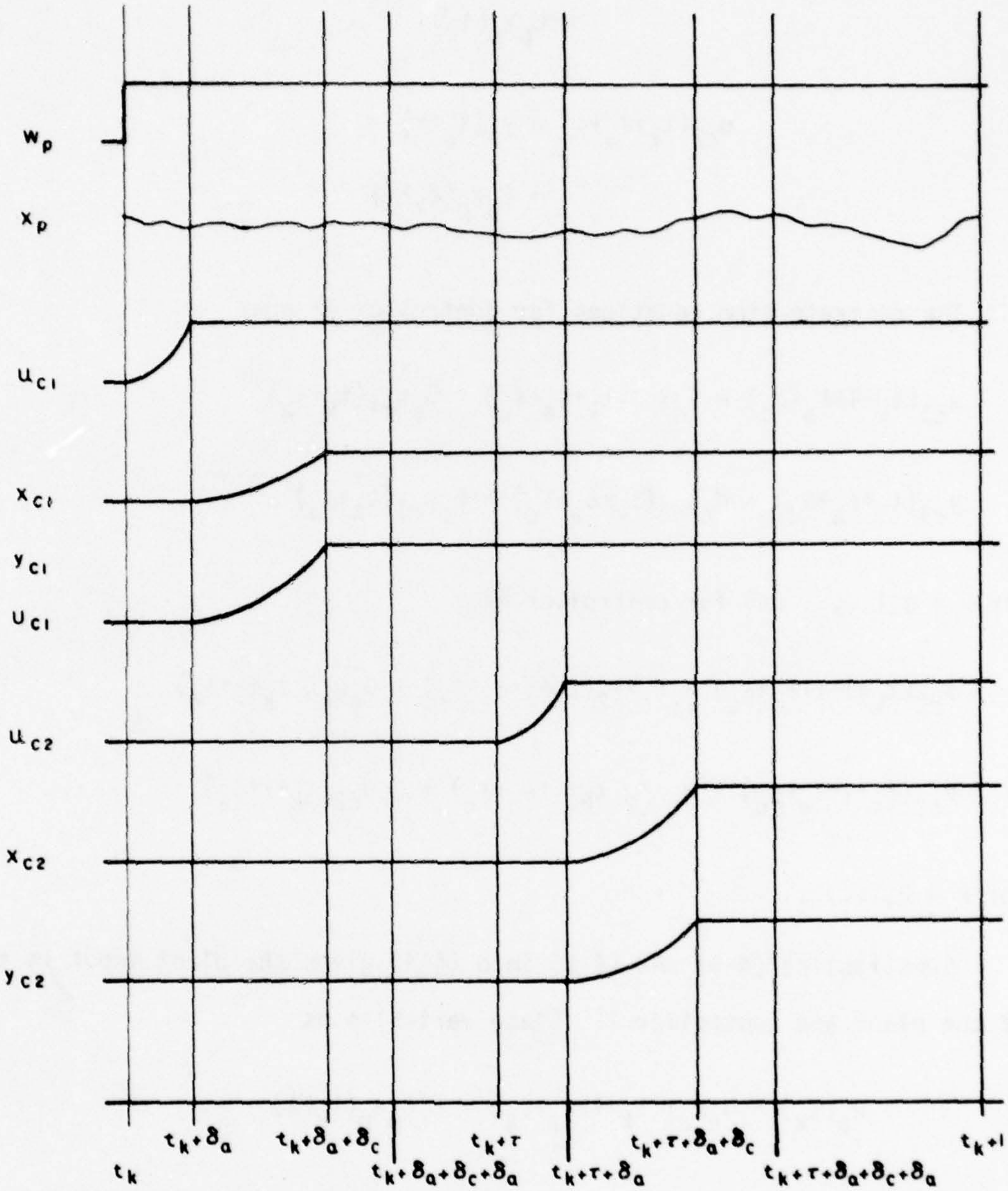
$$y_p = C_p x_p \tag{4-2}$$

for which the solution is

$$x_p(t) = \Phi(t,t_o)x_p(t_o) + \int_{t_o}^{t} \Phi(t,s)B_{1p}u_p(t)ds + \int_{t_o}^{t} \Phi(t,s)B_{2p}w_p(t_o)ds \tag{4-3}$$

As in the basic model, the control input for the plant is the output of the controller 1 and the plant output is the input for each controller at the different times. Thus

$$y_{c1}(t_k) = u_p(t_k) \tag{4-4}$$



$0<\Delta<\tau$ and $\Delta<\tau<T-\Delta$

FIGURE 13 EVENT DIAGRAM OF VIRIATION A

$$u_{c1}(t_k+\delta_a) = y_p(t_k) \tag{4-5}$$
$$= C_p x_p(t_k)$$

$$u_{c2}(t_k+\delta_a+\delta) = y_p(t_k+\tau) \tag{4-6}$$
$$= C_p x_p(t_k+\tau)$$

The discrete-time equations for controller #1 are:

$$x_{c1}(t_k+1+\delta_a+\delta_c) = F_c x_{c1}(t_k+\delta_a+\delta_c) + G_c u_{c1}(t_k+\delta_a) \tag{4-7}$$

$$y_{c1}(t_k+\delta_a+\delta_c) = H_c x_{c1}(t_k+\delta_a+\delta_c) + E_c u_{c1}(t_k+\delta_a) \tag{4-8}$$

for k = 0,1,..., and for controller #2.

$$x_{c2}(t_k+1+\tau+\delta_a+\delta_c) = F_c x_{c2}(t_k+\tau+\delta_a+\delta_c) + G_c u_{c2}(t_k+\tau+\delta_a) \tag{4-9}$$

$$y_{c2}(t_k+\tau+\delta_a+\delta_c) = H_c x_{c2}(t_k+\tau+\delta_a+\delta_c) + E_c u_{c2}(t_k+\tau+\delta_a) \tag{4-10}$$

for k = 0,1,..., .

Substituting (4-5) and (4-8) into (4-4) gives the plant input in terms of the plant and controller 1. State variables as

$$u_p(t_k) = H_c x_{c1}(t_k-1+\delta_a+\delta_c) + E_c C_p x_p(t_k-1) \tag{4-11}$$

Define new variables x_{hp1} and x_{hc1}, in order to have the variables that change in a piecewise constant manner (otherwise we would have had to include $x_p(t_k-1)$ and $x_{c1}(t_k-1+\delta_a+\delta_c)$). There are two additional equations that are required

$$x_{hp1}(t_k+1) = x_p(t_k) \tag{4-12}$$

and

$$x_{hc1}(t_k+1+\delta_a+\delta_c) = x_{c1}(t_k+\delta_a+\delta_c) \tag{4-13}$$

Then (4-11) becomes

$$u_p(t_k) = H_c x_{hc1}(t_k+\delta_a+\delta_c) + E_c C_p x_{hp1}(t_k) \tag{4-14}$$

At time $t_k+\Delta$, a new value of u_p takes effect. So let $t_o=t_k$ and $t = t_k+\Delta$, then (4-1) becomes

$$x_p(t_k+\Delta) = \Phi(\Delta)x_p(t_k) + \psi_1(\Delta)u_p(t_k) + \psi_2(\Delta)w_p(t_k) \tag{4-15}$$

where

$$\psi_1(\Delta) = e^{A\Delta} \int_0^\Delta e^{-A\sigma}B_{1p}d\sigma$$

and

$$\psi_2(\Delta) = e^{A\Delta} \int_0^\Delta e^{-A\sigma}B_{2p}d\sigma$$

Substituting (4-14) with (4-15) gives,

$$x_p(t_k+\Delta) = \Phi(\Delta)x_p(t_k) + \psi_1(\Delta)H_c x_{hc1}(t_k+\delta_a+\delta_c) + \psi_1(\Delta)E_c C_p x_{hp1}(t_k)$$
$$+ \psi_2(\Delta)w_p(t_k) \tag{4-16}$$

At $t = t_k+1$, $x_p(t_k+1)$ depends on the value of $x_p(t_k+\Delta)$, $u_p(t_k+\Delta)$ and $w_p(t_k+\Delta)$. So, let $t_o = t_k+\Delta$ and $t = t_k+1$. The equation (4-1) becomes:

$$x_p(t_k+1) = \Phi(T-\Delta)x_p(t_k+\Delta) + \psi_1(T-\Delta)u_p(t_k+\Delta) + \psi_2(T-\Delta)w_p(t_k) \tag{4-17}$$

By (4-4) and (4-8);

$$u_p(t_k+\Delta) = H_c x_{c1}(t_k+\delta_a+\delta_c) + E_c C_p x_p(t_k) \tag{4-18}$$

Substituting (4-16) and (4-18) into (4-17), gives

$$\begin{aligned} x_p(t_k+1) = {} & [\Phi(T) + \psi_1(T-\Delta)E_c C_p]x_p(t_k) + \Phi(T-\Delta)\psi_1(\Delta)E_c C_p x_{hp1}(t_k) \\ & + \psi_1(T-\Delta)H_c x_{c1}(t_k+\delta_a+\delta_c) + \Phi(T-\Delta)\psi_1(\Delta)H_c x_{hc1}(t_k+\delta_a+\delta_c) \\ & + [\Phi(T-\Delta)\psi_2(\Delta) + \psi_2(T-\Delta)]w_p(t_k) \end{aligned} \tag{4-19}$$

Substituting (4-5) into (4-7) and (4-8) gives

$$x_{c1}(t_k+1+\delta_a+\delta_c) = F_c x_{c1}(t_k+\delta_a+\delta_c) + G_c C_p x_p(t_k) \tag{4-20}$$

and

$$y_{c1}(t_k+\delta_a+\delta_c) = H_c x_{c1}(t_k+\delta_a+\delta_c) + E_c C_p x_p(t_k) \tag{4-21}$$

Similarly for y_{c2} and x_{c2}, substituting (4-6) into (4-9) and (4-10) gives

$$x_{c2}(t_k+1+\tau+\delta_a+\delta_c) = F_c x_{c2}(t_k+\tau+\delta_a+\delta_c) + G_c C_p x_p(t_k+\tau) \tag{4-22}$$

and

$$y_{c2}(t_k+\tau+\delta_a+\delta_c) = H_c x_{c2}(t_k+\tau+\delta_a+\delta_c) + E_c C_p x_p(t_k+\tau) \tag{4-23}$$

The quantity $x_p(t_k+\tau)$ can be written using the solution to equation (4-19) as

$$\begin{aligned} x_p(t_k+\tau) = {} & [\Phi(\tau) + \psi_1(\tau-\Delta)E_c C_p]x_p(t_k) + \Phi(\tau-\Delta)\psi_1(\Delta)E_c C_p x_{hp1}(t_k) \\ & + \psi_1(\tau-\Delta)H_c x_{c1}(t_k+\delta_a+\delta_c) + \Phi(\tau-\Delta)\psi_1(\Delta)H_c x_{hc1}(t_k+\delta_a+\delta_c) \\ & + [\Phi(\tau-\Delta)\psi_2(\Delta) + \psi_2(\tau-\Delta)]w_p(t_k) \end{aligned} \tag{4-24}$$

By substituting (4-24) into (4-22) and (4-23). The controller 2 equations are obtained as

$$x_{c2}(t_k+1+\tau+\delta_a+\delta_c) = G_cC_p[\Phi(\tau) + \psi_1(\tau-\Delta)E_cC_p] + G_cC_p\ (\tau-\Delta)\psi_1(\Delta)E_cC_px_{hp1}(t_k)$$
$$+ G_cC_p\psi_1(\tau-\Delta)H_cx_{c1}(t_k+\delta_a+\delta_c) + G_cC_p\Phi(\tau-\Delta)\psi_1(\Delta)H_cx_{hc1}(t_k+\delta_a+\delta_c)$$
$$+ F_cx_{c2}(t_k+\tau+\delta_a+\delta_c) + G_cC_p[\Phi(\tau-\Delta)\psi_2(\Delta) + \psi_2(\tau-\Delta)]w_p(t_k) \quad (4\text{-}25)$$

and

$$y_{c2}(t_k+\tau+\delta_a+\delta_c) = E_cC_p[\Phi(\tau) + \psi_1(\tau-\Delta)E_cC_p] + E_cC_p\Phi(\tau-\Delta)\psi_1(\Delta)E_cC_px_{hp1}(t_k)$$
$$+ E_cC_p\psi_1(\tau-\Delta)H_cx_{c1}(t_k+\delta_a+\delta_c) + E_cC_p\Phi(\tau-\Delta)\psi_1(\Delta)H_cx_{hc1}(t_k+\delta_a+\delta_c)$$
$$+ F_cx_{c2}(t_k+\tau+\delta_a+\delta_c) + E_cC_p[\Phi(\tau-\Delta)\psi_2(\Delta) + \psi_2(\tau-\Delta)]w_p(t_k) \quad (4\text{-}26)$$

The inherent error is defined in two parts as follows:

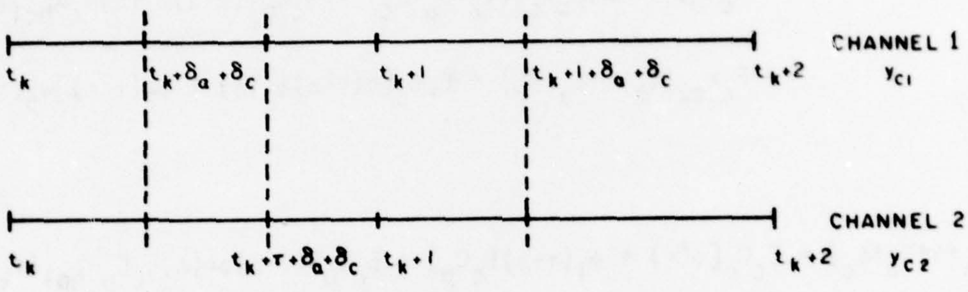
$$e_A(t) = y_{c1}(t_k+\delta_a+\delta_c) - y_{c2}(t_k+\tau+\delta_a+\delta_c) \quad (4\text{-}27)$$

for $t_k+\tau+\delta_a+\delta_c \leq t < t_k+1+\delta_a+\delta_c$, $k = 0,1,\ldots,$ $0\leq\tau<T-\Delta$ and

$$e_B(t) = y_{c1}(t_k+1+\delta_a+\delta_c) - y_{c2}(t_k+\tau+\delta_a+\delta_c) \quad (4\text{-}28)$$

for $t_k+1+\delta_a+\delta_c \leq t < t_k+1+\tau+\delta_a+\delta_c$, $k = 0,1,\ldots,$ $\Delta<\tau\leq T-\Delta$.

Figure 14 shows the skewed sampling and inherent errors of the Delay Model. Channel 1 produces the sampled outputs at times $t_k+\delta_a+\delta_c$, $t_k+1+\delta_a+\delta_c,\ldots,$ for $k = 0,1,\ldots,$ and channel 2 produces the sampled outputs at times $t_k+\tau+\delta_a+\delta_c$, $t_k+1+\tau+\delta_a+\delta_c\ldots.$.



$$\varepsilon_A(t) = y_{C1}(t_k+\delta_a+\delta_c) - y_{C2}(t_k+\tau+\delta_a+\delta_c)$$

FOR $t_k+\tau+\delta_a+\delta_c \le t < t_k+1+\delta_a+\delta_c \qquad k=0,1\ldots,\ \Delta \le \tau < T-\Delta$

$$\varepsilon_B(t) = y_{C1}(t_k+1+\delta_a+\delta_c) - y_{C2}(t_k+\tau+\delta_a+\delta_c)$$

FOR $t_k+1+\delta_a+\delta_c \le t < t_k+1+\tau+\delta_a+\delta_c \qquad k=0,1\ldots,\ \Delta < \tau \le T-\Delta$

WHERE $\Delta = \delta_a+\delta_c+\delta_d$

FIGURE 14 SKEWED SAMPLING AND INHERENT ERRORS

These equations can be put in compact form by writing them in terms of a combined stated vector.

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{hp1}(t_k) \\ x_{c1}(t_k+\delta_a+\delta_c) \\ x_{hc1}(t_k+\delta_a+\delta_c) \\ x_{c2}(t_k+\tau+\delta_a+\delta_c) \end{bmatrix} \quad \text{and} \quad x(t_k+1) = \begin{bmatrix} x_p(t_k+1) \\ x_{hp1}(t_k+1) \\ x_{c1}(t_k+1+\delta_a+\delta_c) \\ x_{hc1}(t_k+1+\delta_a+\delta_c) \\ x_{c2}(t_k+1+\tau+\delta_a+\delta_c) \end{bmatrix}$$

The state equations become

$$x(t_k+1) = F(T,\tau)\ x(t_k) + G(T,\tau)\ w_p(t_k) \tag{4-29}$$

where $F(T,\tau)$ is

$$f_{11} = \Phi(T) + \psi_1(T-\Delta)\ E_c C_p$$
$$f_{12} = \Phi(T-\Delta)\ \psi_1(\Delta)\ E_c C_p$$
$$f_{13} = \psi_1(T-\Delta)\ H_c$$
$$f_{14} = \Phi(T-\Delta)\psi_1(\Delta)H_c$$
$$f_{15} = 0$$
$$f_{21} = 1$$
$$f_{22} = \delta_{23} = \delta_{24} = \delta_{25} = 0$$
$$f_{31} = G_c C_p$$
$$f_{32} = 0$$
$$f_{33} = F_c$$
$$f_{34} = \delta_{35} = 0$$
$$f_{41} = \delta_{42} = 0$$
$$f_{43} = 1$$
$$f_{44} = \delta_{45} = 0$$
$$f_{51} = G_c C_p[\Phi(\tau) + \psi_1(\tau-\Delta)E_c C_p]$$
$$f_{52} = G_c C_p \Phi(\tau-\Delta)\psi_1(\Delta)\ E_c C_p$$

$$f_{53} = G_cC_p\ \psi_1(\tau-\Delta)H_c$$
$$f_{56} = G_cC_p\Phi\ (\tau-\ )\psi_1(\Delta)H_c$$
$$f_{57} = F_c$$

and $G(T,\tau)$ is

$$g_1 = \ (T-\ )\psi_2(\Delta)\ +\psi_2(T-\Delta)$$
$$g_2 = g_3 = g_4 = 0$$
$$g_5 = G_cC_p[\ (\tau-\Delta)\psi_2(\Delta) + \psi_2(\tau-\Delta)]$$

The controller output equations are

$$y_{c1}(t_k+\delta_a+\delta_c) = H_1x(t_k) \tag{4-30}$$

and

$$y_{c2}(t_k+\tau+\delta_a+\delta_c) = H_2x(t_k) + \rho\ w_p(t_k) \tag{4-31}$$

where

$$H_1 = [E_cC_p \quad 0 \quad H_c \quad 0 \quad 0] \tag{4-32}$$

and

$$H_2 = [E_cC_p(\Phi(\tau) + \psi_1(\tau-\Delta)E_cC_p)\ E_cC_p\Phi(\tau-\Delta)\psi_1(\Delta)E_cC_p\ E_cC_p\psi_1(\tau-\Delta)H_c$$
$$E_cC_p\Phi(\tau-\Delta)\psi_1(\Delta)H_c\ \ H_c] \tag{4-33}$$

$$\rho = E_cC_p[\Phi(\tau-\Delta)\psi_2(\Delta) + \psi_2(\tau-\Delta)] \tag{4-34}$$

The expressions of $e_A(t)$ and $e_B(t)$ become

$$e_A(t) = (H_1-H_2)\ x(t_k) - \rho\ w_p(t_k) \tag{4-35}$$

for $t_k+\tau+\delta_a+\delta_c \leq t < t_k+1+\delta_a+\delta_c$, $k = 0,1,\ldots,\ \Delta \leq\tau<T-\Delta$ and

$$e_B(t) = [H_1F(T,\tau) - H_2]x(t_k) + [H_1G(T,\tau)-\rho]w_p(t_k) \tag{4-36}$$

for $t_k+1+\delta_a+\delta_c \leq t < t_k+1+T+\delta_a+\delta_c$, $k = 0,1,\ldots,$ $\Delta<\tau\leq T-\Delta$

Let E_A be the average error, so

$$E_A = \frac{1}{2}[e_A(t) - e_B(t)]$$

$$= \frac{1}{2}[H_1+H_1F(T,\tau) - 2H_2]\ x(t_k) + \frac{1}{2}[H_1G(T,\tau) - 2\rho]w_p(t_k) \tag{4-37}$$

4.1.2 Covariance Analysis

As in the multirate model, the input $w_p(t_k)$ is a Gaussian white noise random process with zero mean which is independent of $x(0)$. Then (3-54), (3-55) and (3-56) are repeated here as

$$E[w_p(t_k)] = 0 \tag{4-38}$$

$$E[x(t_k)w_p^T(t_k)] = 0 \tag{4-39}$$

$$E[w_p(t_k)w_p^T(t_k)] = w_k \tag{4-40}$$

The covariance matrix of the states is defined as

$$P_x(k) = E[x(t_k)x^T(t_k)] \tag{4-41}$$

and

$$P_x(k+1) = E[x(t_k+1)x^T(t_k+1)] \tag{4-42}$$

$$= F(T,\tau)P_x(k)F^T(T,\tau) + G(T,\tau)W_kG^T(T,\tau) \tag{4-43}$$

The steady-state covariance, designated P_{xss} is found by solving the equation

$$P_{xss} = F(T,\tau)\ P_{xss}\ F^T(T,\tau) + G(T,\tau)\ w_kG^T(T,\tau) \tag{4-44}$$

Covariance of the Errors

The covariances of $e_A(t)$ and $e_B(t)$ are calculated using the same procedure as in the previous development. Let P_{eA} be the covariance of e_A, then

$$P_{eA}(t) = E[e_A(t)e_A{}^T(t)] \tag{4-45}$$

for $t_k+\tau+\delta_a+\delta_e \leq t < t_k+1+\delta_a+\delta_c$, $k = 0,1,\ldots$, $\Delta\leq\tau<T-\Delta$

Substituting (4-35) with (4-45) gives

$$P_{eA}(t) = H_A P_x(k) H_A{}^T + \rho w_k \rho^T \tag{4-46}$$

for $t_k+\tau+\delta_a+\delta_c \leq t < t_k+1+\delta_a+\delta_c$, $k = 0,1,\ldots$, $\Delta\leq\tau<T-\Delta$

where

$$H_A = H_1 - H_2 \tag{4-47}$$

Let P_{eB} be the covariance of e_B, then

$$P_{eB}(t) = E[e_B(t)e_B{}^T(t)] \tag{4-48}$$

for $t_k+1+\delta_a+\delta_c \leq t < t_k+1+\tau+\delta_a+\delta_c$, $k = 0,1,\ldots$, $\Delta<\tau\leq T-\Delta$

Substituting (4-36) into (4-48) gives

$$P_{eB}(t) = H_B P_x(k) H_B{}^T + e_B w_k e_B{}^T \tag{4-49}$$

for $t_k+1+\delta_a+\delta_c \leq t < t_k+1+\tau+\delta_a+\delta_c$, $k = 0,1,\ldots$, $\Delta<\tau\leq T-\Delta$

where

$$H_B = H_1 F(T,\tau) - H_2 \tag{4-50}$$

and

$$\rho_B = H_1 G(T,\tau) - \rho \tag{4-51}$$

From Figure 12, if the value of the time delays $(\delta_a, \delta_c, \delta_d)$ are equal to zero. The model is close to the basic model except that the input w_p in this model is sampled and zero-order hold. Then if the time delays of this variation are equal to zero, the equations of this variation should be close to the equations of the basic model or the equations of the multirate at n=1.

Let Δ=0, then the term x_{hp1} and x_{hc1} become x_p and x_{c1} or $x(t_k)$ and $x(t_k+1)$ of this variation become

$$x(t_k) = \begin{bmatrix} x_p(t_k) + x_{hp1}(t_k) \\ x_{c1}(t_k) + x_{hc1}(t_k) \\ x_{c2}(t_k+\tau) \end{bmatrix}, \text{ and } x(t_k+1) = \begin{bmatrix} x_p(t_k+1) + x_{hp1}(t_k+1) \\ x_{c1}(t_k+1) + x_{hc1}(t_k+1) \\ x_{c2}(t_k+1+\tau) \end{bmatrix}$$

Then $F(T,\tau)$ and $G(T,\tau)$ becomes

$$F(T,\tau) = \begin{bmatrix} \Phi(T) + \psi_1(T)E_cC_p & \psi_1(T)H_c & 0 \\ G_cC_p & F_c & \\ G_cC_p[\Phi(\tau) + \psi_1(\tau)E_cC_p] & G_cC_p\psi_1(\tau)H_c & 0 \end{bmatrix}$$

$$G(T,\tau) = \begin{bmatrix} \Psi_2(T) \\ 0 \\ G_cC_p\ {}_2(\tau) \end{bmatrix}$$

where $\psi_1(\delta_c) = \psi_2(\delta_c) = 0$.

The matrix $F(T,\tau)$ is identically to $F(T,\tau)$ in the Basic Model and the matrix $G(T,\tau)$ is close to the second matrix of $x(t_k+1)$ in the Basic Model. This is because the noise of this model is sampled and zero-order hold but the noise of the Basic Model is continuous. Now, examine H_1 and H_2, with Δ=0. The

equations of H_1 and H_2 form the equations (4-32) and (4-33) become

$$H_1 = [E_cC_p \quad H_c \quad 0]$$

and

$$H_2 = [E_cC_p[\Phi(\tau) + \psi_1(\tau)E_cC_p] \; E_cC_p\psi_1(\tau)H_c \quad F_c]$$

These two matrices are the same as H_1 and H_2 in the basic model. By this technique, the equations of this variation (time delay = 0) are close to the equations of the basic model and they are the same as the equations of the multirate model as expected. By using the data from the first example of section 2 (Basic Model), we can study the characteristic of the covariance errors of this variation. As discussed above, with data from the first example of section 2, if we let the time delays in the expressions of the covariance errors, the results must be equal to the covariance errors of the multirate model (n=1).

4.1.3 Example

The data from the first example of section II are

$A_p = 0$	$F_c = 0$
$B_{1p} = 1$	$G_c = 0$
$B_{2p} = 1$	$H_c = 0$
$C_p = 1$	$E_c = -k$

Let $w_k = \sigma_w^2/T$.

$\Phi(t,t_o) = 1$

$\psi_1(T) = \psi_2(T) = T$

$\psi_1(T-\Delta) = \psi_2(T-\Delta) = T-\Delta$

$\psi_1(\tau) = \psi_2(\tau) = \tau$

$\psi_1(\tau-\Delta) = \psi_2(\tau-\Delta) = \tau-\Delta$

By using the equations of matrices $F(T,\tau)$ and $G(T,\tau)$ with these data, we have

$$F = \begin{bmatrix} 1-k(T-\Delta) & -k\Delta & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-52}$$

$$G = \begin{bmatrix} T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4-53}$$

The steady state covariance of the states if found by solving (4-44)

$$P_{xss} = \begin{bmatrix} \frac{\sigma_w^2 T}{1-(1-kT)^2} & 0 & 0 & \frac{\sigma_w^2 T}{1-(1-kT)(1-kT)} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-54}$$

By using equations (4-32), (4-33) and (4-34) we have

$$H_1 = [-k \quad 0 \quad 0 \quad 0 \quad 0] \tag{4-55}$$

$$H_2 = [-k+k^2(\tau-\Delta) \quad k^2\Delta \quad 0 \quad 0 \quad 0] \tag{4-56}$$

$$\rho = -k\tau \tag{4-57}$$

Therefore we get

$$P_{eAss} = \frac{k^3(\tau-\Delta)^2\sigma_w{}^2}{(2-kT)} + \frac{k^2\tau^2\sigma_w{}^2}{T} \quad (4\text{-}58)$$

and

$$P_{eBss} = k^2(T-\tau)\sigma_w{}^2\left[\frac{k(T-\tau)}{2-kT} + \frac{(T-\tau)}{T}\right] \quad (4\text{-}59)$$

Let $\Delta=0$. Then we have the expression of P_{eAss} and P_{eBss} of the Basic model with discrete noise.

$$P_{eAss1} = k^2\tau\sigma_w{}^2\left[\frac{k\tau}{2-kT} + \frac{\tau}{T}\right] \quad (4\text{-}60)$$

and

$$P_{eBss1} = k^2(T-\tau)\sigma_w{}^2\left[\frac{k(T-\tau)}{2-kT} + \frac{(T-\tau)}{T}\right] \quad (4\text{-}61)$$

These covariance errors are equal to the covariance errors of the multirate model when n=1 (equations (3-83) and (3-84)) as expected. As the same reason in the multirate model, the covariance errors of this variation are the approximate values and they are less than the true value.

P_{eAss} (4-58) and P_{eBss} (4-59) are plotted in Figure 15 as a function of τ. The diagrams to the right of each plot show the times corresponding to the value of the controller outputs used to calculate e and e_B. From (4-58) and (4-59), P_{eAss} depends on the value of the time delays but P_{eBss} doesn't. Under the specific case of the zero value of A_p makes P_{eBss} does not depend on the time delay, but for other values of A_p both P_{eAss} and P_{eBss} depend on the time delays. However, in this example, the results obtained for this example agree with intuition. For small τ, the variance of e_A is small and that of e_B is large. For large τ, the complimentary situation holds.

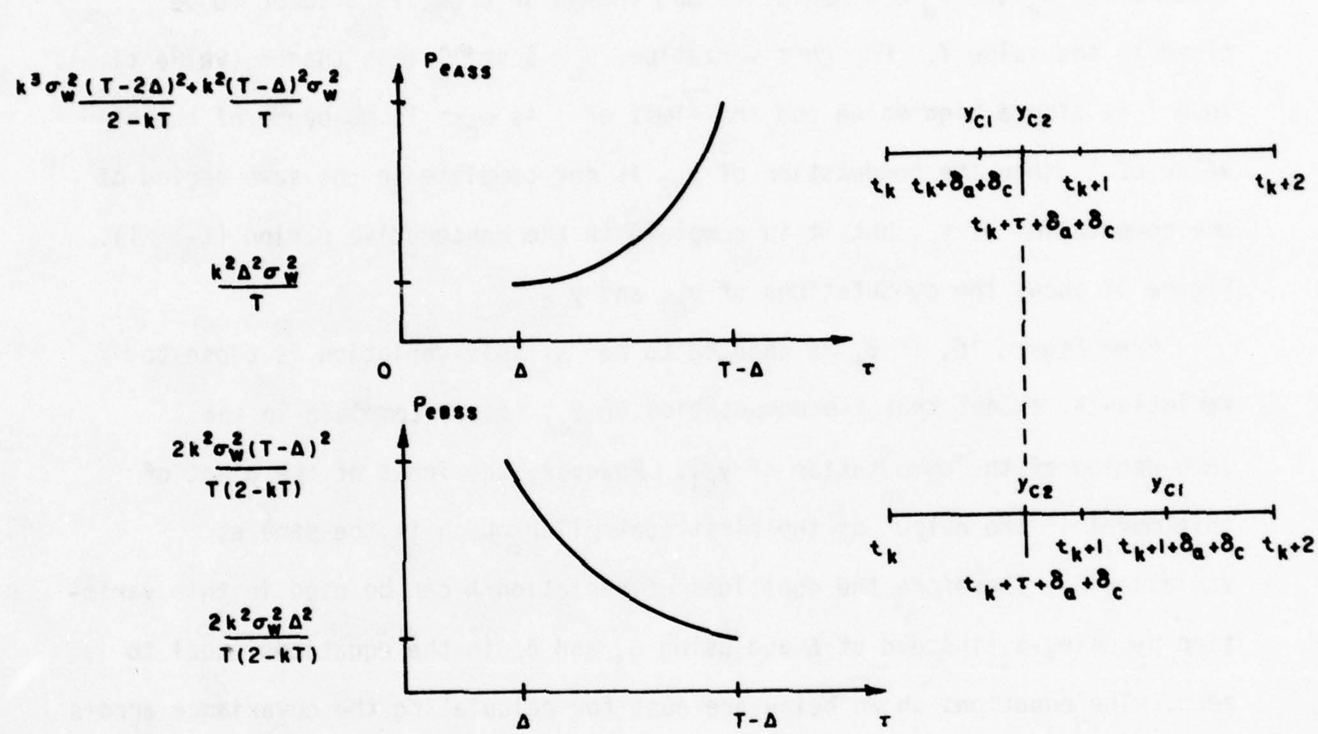


FIGURE 15 P_{eASS} and P_{eBSS} $\Delta<\tau<T-\Delta$

4.2 Variation B

Physically, the value of time delay at the controller (δ_c) is a high value and it is greater than the values of time delay at the A/D converter (δ_a) and at the D/A converter (δ_d). Then in this variation and the next variation of this model, δ_a and δ_d are neglected and the value of δ_c is assumed to be close to the value T. For this variation, δ_c is still less than τ (value time). Then τ is also a high value and the limit of τ is $\delta_c<\tau<T$. Because of the high value of δ_c then the computation of y_{c2} is not complete in the same period of the computation of y_{c1} but it is complete in the consecutive period ($\tau+\delta_c>T$). Figure 16 shows the computations of y_{c1} and y_{c2}.

From Figure 16, if δ_c is changed to be Δ, this variation is close to variation A, except that the computation of y_{c2} is not complete in the same period of the computation of y_{c1}. However, the input of the plant of this model is the output of the first controller which is the same as variation A. Therefore the equations of variation A can be used in this variation by using δ_c instead of Δ and using δ_a and δ_c in the equations equal to zero. The equations shown below are just for calculating the covariance errors of this variation.

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{wp}(t_k) \\ x_{c1}(t_k+\delta_c) \\ x_{hc1}(t_k+\delta_c) \\ x_{c2}(t_k+\tau+\delta_c) \end{bmatrix}, x(t_k+1) = \begin{bmatrix} x_p(t_k+1 \\ x_{hp1}(t_k+1) \\ x_{c1}(t_k+1+\delta_c) \\ x_{hc1}(t_k+1+\delta_c) \\ x_{c2}(t_k+1+\tau+\delta_c) \end{bmatrix}$$

and

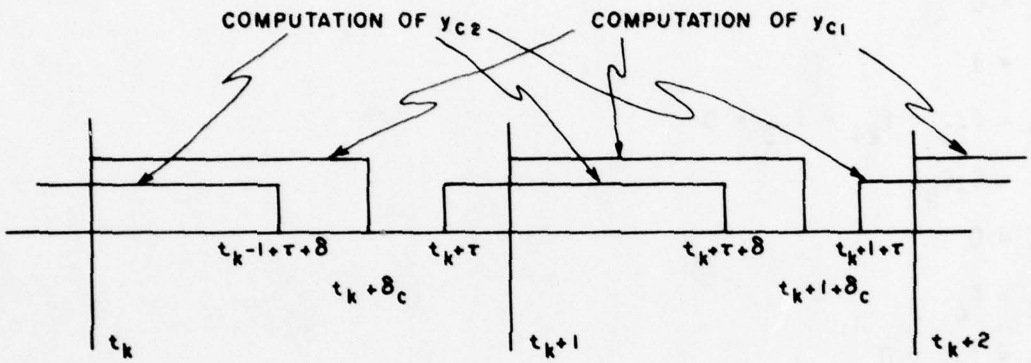


FIGURE 16 THE COMPUTATION OF y_{C1} and$_{C2}$

$$x(t_k+1) = F(T,\tau)x(t_k) + G(T,\tau)\ w_p(t_k)$$

where $F(T,\tau)$ is

$f_{11} = \Phi(T) + \psi_1(T\ \delta_c)E_cC_p$

$f_{12} = \Phi(t-\delta_c)\psi_1(\delta_c)E_cC_p$

$f_{13} = \psi_1(T-\delta_c)H_c$

$f_{14} = \Phi(T-\delta_c)\psi_1(\delta_c)H_c$

$f_{15} = 0$

$f_{21} = 1$

$f_{22} = f_{23} = f_{24} = f_{25} = 0$

$f_{31} = G_cC_p$

$f_{32} = 0$

$f_{33} = F_c$

$f_{34} = f_{35} = 0$

$f_{41} = f_{42} = 0$

$f_{43} = 1$

$f_{44} = f_{45} = 0$

$f_{51} = G_cC_p[\Phi(\tau) + \psi_1(\tau-\delta_c)E_cC_p]$

$f_{52} = G_cC_p\Phi(\tau-\delta_c)\psi_1(\delta_c)E_cC_p$

$f_{53} = G_cC_p\psi_1(\tau-\delta_c)H_c$

$f_{54} = G_cC_p\Phi(\tau-\delta_c)\psi_1(\tau)H_c$

$f_{55} = F_c$

and $G(T,\tau)$ is

$g_1 = \Phi(T-\delta_c)\psi_2(\delta_c) + \psi_2)T-\delta_c)$

$g_2 = g_3 = g_4 = 0$

$g_5 = G_cC_p\ [\Phi(\tau-\delta_c)\psi_2(\delta_c) + \psi_2(\tau-\delta_c)]$

The controller output equations are

$$y_{c1}(t_k+\delta_c) = H_1 x(t_k)$$

and

$$y_{c2}(t_k+\tau+\delta_c) = H_2 x(t_k) + \rho\, w_p(t_k)$$

where

$$H_1 = [E_c C_p \quad 0 \quad H_c \quad 0 \quad 0]$$

and

$$H_2 = [E_c C_p(\Phi(\tau) + \psi_1(\tau-\delta_c)E_c C_p) \;\; [E_c C_p \Phi(\tau-\delta_c)\psi_1(\delta_c) \;\; E_c C_p$$

$$E_c C_p(\psi_1(\tau-\delta_c)H_c \quad E_c C_p \Phi(\tau-\delta_c)\psi_1(\delta_c)H_c \quad H_c]$$

$$\rho = E_c C_p[\Phi(\tau-\delta_c)\psi_2(\delta_c) + \psi_2(\tau-\delta_c)]$$

$$e_A(t) = (H_1-H_2)\, x\,(t_k) - \rho\, w_p(t_k)$$

for $t_k+\tau+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,$ $\Delta \leq \tau < T$ and

$$e_B(t) = (H_1 F(T,\tau) - H_2)\, x(t_k) + [H_1 G(T,\tau) - \rho] w_p(t_k)$$

for $t_k+1+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,$ $\Delta < \tau \leq T$. The average error of e_A and e_B are

$$E_A = \frac{1}{2}\,[e_A(t) + e_B(t)]$$

The covariance of states

$$P_x(k+1) = F(T,\tau)\, P_x(k)\, F^T(T,\tau) + G(T,\tau) w_k G^T(T,\tau)$$

The covariance of errors

$$P_{eA}(t) = H_A P_x(k) H_A{}^T + \rho w_k \rho^T$$

for $t_k+1+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,\ldots,$ $\Delta \leq \tau < T$.

where

$$H_A = H_1 - H_2$$

and

$$P_{eA}(t) = H_B P_x(k)\, H_B^T + \rho_B w_k \rho_B^T$$

for $t_k+1+\delta_c \leq t < t_k+1 + \tau + \delta_c$, $k = 0,1,\ldots,$ $\Delta<\tau\leq T$

where $H_B = H_1 F - H_2$

and

$$\rho_B = H_1 G - \rho$$

Recall the expressions of P_{eAss} and P_{eBss} from the example in variation A, we have

$$P_{eAss} = k^3 \frac{(\tau-\delta_c)^2 \sigma_w^2}{2-kT} + \frac{k^2 T^2 \sigma_w^2}{T}$$

for $t_k+\tau+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,\ldots,$ $\delta_c \leq \tau < T$. And

$$P_{eBss} = k^2 (T-\tau) \sigma_w^2 \left[\frac{k(T-\tau)}{2-kT} + \frac{(T-\tau)}{T}\right]$$

for $t_k+1+\delta_c \leq t$ $t_k+1+\tau+\delta_c$, $k = 0,1,\ldots,\delta_c < \tau \leq T$.

P_{eAss} and P_{eBss} are plotted in Figure 17 as a function of τ. The diagrams to the right of each plot show the times corresponding to the value of the controller outputs used to calculate e_A and e_B. Variation A, P_{eBss} depends on the time delay δ_c but P_{eBss} does not. The results obtained for this example agree with intuition. For small τ, the variance of e_A is small and that of e_B is large. For large τ, the complimentary situation holds.

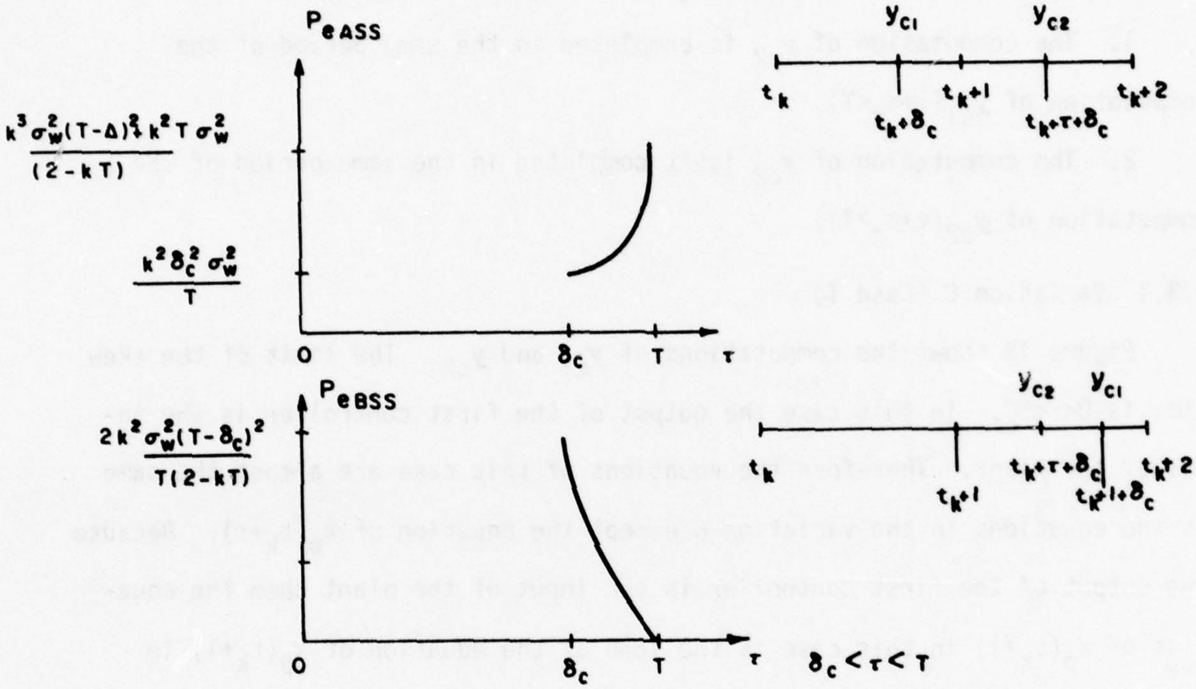


FIGURE 17 PeASS and PeBSS

4.3 Variation C

The same as variation B, we have only one time delay δ_c but in this variation, the value of δ_c is greater than the value of other time (τ). There are two cases of this variation:

1. The computation of y_{c2} is completed in the same period of the computation of $y_{c1}(\tau+\delta_c<T)$.

2. The computation of y_{c2} isn't completed in the same period of the computation of $y_{c2}(\tau+\delta_c>T)$.

4.3.1 Variation C (Case I)

Figure 18 shows the computations of y_{c1} and y_{c2}. The limit of the skew time is $0<\tau<\delta_c$. In this case the output of the first controller is the input of the plant. Therefore the equations of this case are almost the same as the equations in the variation B except the equation of $x_p(t_k+\tau)$. Because the output of the first controller is the input of the plant then the equation of $x_p(t_k+1)$ in this case is the same as the equation of $x_p(t_k+1)$ in variation B and from variation B.

$$\begin{aligned} x_p(t_k+1) = {} & [\Phi(T) + \psi_1(T-\delta_c)E_cC_p]x_p(t_k) + \Phi(T-\delta_c)\psi_1(\delta_c)E_cC_px_{hp1}(t_k) \\ & + \psi_1(T-\delta_c)H_cx_{c1}(t_k+\delta_c) + \Phi(T-\delta_c)\psi_1(\delta_c)H_cx_{hc1}(t_k+\delta_c) \\ & + [\Phi(T-\delta_c)\psi_2(\delta_c) + \psi_2(T-\delta_c)]\ w_p(t_k) \end{aligned}$$

The controller equations are

$$x_{c1}(t_k+1+\delta_c) = F_cx_{c1}(t_k+\delta_c) + G_cC_px_p(t_k)$$

$$y_{c1}(t_k+\delta_c) = H_cx_{c1}(t_k+\delta_c) + E_cC_px_p(t_k)$$

$$x_{c2}(t_k+1+\tau+\delta_c) = F_cx_{c2}(t_k+\tau+\delta_c) + G_cC_px_p(t_k+\tau)$$

and

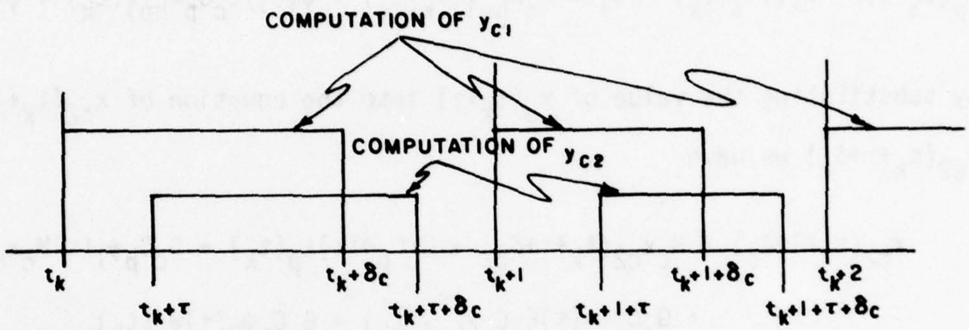


FIGURE 18 THE COMPUTATIONS OF y_{C1} and y_{C2}

$$y_{c2}(t_k+\tau+\delta_c) = H_c x_{c2}(t_k+\tau+\delta_c) + E_c C_p x_p(t_k+\tau)$$

In this case, the equation $x_p(t_k+\tau)$ is derived by letting $t_o=t_k$ and $t=t_k+\tau$. Then from the plant equation:

$$x_p(t_k+\tau) = \Phi(\tau)x_p(t_k) + \psi_1(\tau)\ u_p(t_k) + \psi_2(\tau)\ w_p(t_k)$$

Substituting $u_p(t_k)$ into the equation of $x_p(t_k+\tau)$ gives

$$x_p(t_k+\tau) = \Phi(\tau)x_p(t_k) + \psi_1(\tau)H_c x_{hc1}(t_k+\delta_c) + \psi_1(\tau)E_c C_p x_{hp1}(t_k) + \psi_2(\tau)w_p(t_k)$$

By substituting the value of $x_p(t_k+\tau)$ into the equation of $x_{c2}(t_k+1+\delta_c+\tau)$ and $y_{c2}(t_k+\tau+\delta_c)$ we have

$$\begin{aligned} x_{c2}(t_k+1+\delta_c) = {} & H_c x_{c2}(t_k+\tau+\delta_c) + G_c C_p \Phi(\tau)x_p(t_k) + G_c C_p \psi_1(\tau)H_c x_{hc1}(t_k+\delta_c) \\ & + G_c C_p \psi_1(\tau)E_c C_p x_{hp1}(t_k) + G_c C_p \psi_2(\tau)w_p(t_k) \end{aligned}$$

and

$$\begin{aligned} y_{c2}(t_k+\tau+\delta_c) = {} & H_c x_{c2}(t_k+\tau+\delta_c) + E_c C_p \Phi(\tau)x_p(t_k) + E_c C_p \psi_1(\tau)H_c x_{hc1}(t_k+\delta_c) \\ & + E_c C_p \psi_1(\tau)E_c C_p x_{hp1}(t_k) + E_c C_p \psi_2(\tau)w_p(t_k). \end{aligned}$$

Let

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{hp1}(t_k) \\ x_{c1}(t_k+\delta_c) \\ x_{hc1}(t_k+\delta_c) \\ x_{c2}(t_k+\tau+\delta_c) \end{bmatrix} \quad \text{and } x(t_k+1) = \begin{bmatrix} x_p(t_k+1) \\ x_{hp1}(t_k+1) \\ x_{c1}(t_k+1+\delta_c) \\ x_{hc1}(t_k+1+\delta_c) \\ x_{c2}(t_k+1+\tau+\delta_c) \end{bmatrix}$$

The states equation is

$$x(t_k+1) = F(T,\tau)\ x(t_k) + G(T,\tau)\ w_p(t_k)$$

Then $F(T,\tau)$ is

$$f_{11} = \Phi(T) + \psi_1(T-\delta_c)E_cC_p$$
$$f_{12} = \Phi(T-\delta_c)\psi_1(\delta_c)E_cC_p$$
$$f_{13} = \psi_1(T-\delta_c)H_c$$
$$f_{14} = \Phi(T-\delta_c)\psi_1(\delta_c)H_c$$
$$f_{15} = 0$$
$$f_{21} = 1$$
$$f_{22} = f_{23} = f_{24} = f_{25} = 0$$
$$f_{31} = G_cC_p$$
$$f_{32} = 0$$
$$f_{33} = F_c$$
$$f_{34} = f_{35} = 0$$
$$f_{41} = f_{42} = 0$$
$$f_{43} = 1$$
$$f_{44} = f_{45} = 0$$
$$f_{51} = G_cC_p\Phi(\tau)$$
$$f_{52} = G_cC_p\psi_1(\tau)E_cC_p$$
$$f_{53} = 0$$
$$f_{54} = G_cC_p\psi_1(\tau)H_c$$
$$f_{55} = F_c$$

and $G(T,\tau)$ is

$$g_1 = \Phi(T-\delta_c)\psi_2(\delta_c) + \psi_2(T-\delta_c)$$
$$g_2 = g_3 = g_4 = 0$$
$$g_5 = G_cC_p\psi_2(\tau)$$

The controller output equations are

$$y_{c1}(t_k+\delta_c) = H_1x(t_k)$$

and

$$y_{c2}(t_k+\tau+\delta_c) = H_2x(t_k) + \sigma w_p(t_k)$$

where

$$H_1 = [E_cC_p \quad 0 \quad H_c \quad 0 \quad 0]$$

and

$$H_2 = [E_cC_p\Phi(\tau) \;\; E_cC_p\psi_1(\tau)E_cC_p \quad 0 \quad E_cC_p\psi_1(\tau)H_c \quad H_c]$$

The inherent errors in this case is defined the same as variation B. Then from variation B

$$e_A(t) = (H_1-H_2)x(t_k) - \sigma w_p(t_k)$$

for $t_k+\tau+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,\ldots,$ $0\leq\tau<\delta_c$

and

$$e_A(t) = (H_1F-H_2)x(t_k) - (H_1G-\sigma)w_p(t_k)$$

for $t_k+1+\delta_c \leq t < t_k+1+\tau+\delta_c$, $k = 0,1,\ldots,$ $0<\tau\leq\delta_c$.

Figure 19 shows the skew sampling and inherent errors of this variation. The average error is

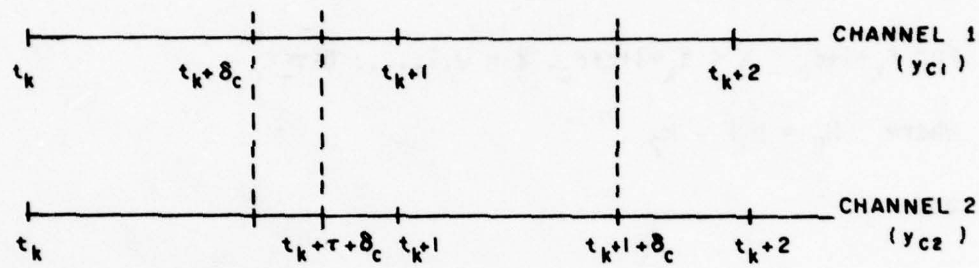
$$E_A = \frac{1}{2}[e_A(t) + e_B(t)]$$

The same as variation B, the covariance of states is

$$P_x(k+1) = F(T,\tau)P_x(k)F^T(T,\tau) + G(T,\tau)w_pG^T(T,\tau)$$

and the steady state covariance is

$$P_{xss} = F(T,\tau)P_{xss}F^T(T,\tau) + G(T,\tau)w_kG^T(T,\tau)$$



$$e_A(t) = y_{c1}(t_k+\delta_c) - y_{c2}(t_k+\tau+\delta_c)$$

FOR $\quad t_k+\tau+\delta_c \le t < t_k+1+\delta_c \qquad k=0,1\ldots, \quad 0 \le \tau < \delta_c$

$$e_B(t) = y_{c1}(t_k+1+\delta_c) - y_{c2}(t_k+\tau+\delta_c)$$

FOR $\quad t_k+1+\delta_c \le t < t_k+1+\tau+\delta_c \qquad k=0,1,\ldots, \quad 0 < \tau \le \delta_c$

FIGURE 19 SKEWED SAMPLING AND INHERENT ERRORS

The equations of the covariance errors are

$$P_{eA}(t) = H_A P_x(k) H_A^T + \rho w_k \rho^T$$

for $t_k+\tau+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,\ldots,$ $0\leq\tau<\delta_c$

where $H_A = H_1 - H_2$ and

$$P_{eB}(t) = H_B P_x(k) H_B^T + \rho_B w_k \rho_B^T$$

for $t_k+1+\delta_c \leq t < t_k+1+\tau+\delta_c$, $k = 0,1,\ldots,$ $0<\tau\leq\delta_c$

where $H_B = H_1 F - H_2$

and

$$\rho_B = H_1 G - \rho$$

The same as the variations A and B, if δ_c is equal to zero, the equations of this case must be equal to the equations of variation A or B which the time delays of these variations are equal to zero. If δ_c is equal to zero then x_{hp1} and x_{hc1} become to x_p and x_{c1} and $x(t_k)$ and $x(t_k+1)$ becomes

$$x(t_k) = \begin{bmatrix} x_p(t_k) + x_{hp1}(t_k) \\ x_{c1}(t_k) + x_{hc1}(t_k) \\ x_{c2}(t_k+\tau) \end{bmatrix} \quad \text{and } x(t_k+1) = \begin{bmatrix} x_p(t_k+1) + x_{hp1}(t_k+1) \\ x_{c1}(t_k+1) + x_{hc1}(t_k+1) \\ x_{c2}(t_k+1+\tau) \end{bmatrix}$$

and the matrices $F(T,\tau)$ and $G(T,\tau)$ are

$$F = \begin{bmatrix} \Phi(T) + \psi_1(T)E_cC_p & \psi_1(T)H_c & 0 \\ G_cC_p & F_c & 0 \\ G_cC_p[\Phi(\tau) + \psi_1(\tau)E_cC_p] & G_cC_p\psi_1(\tau)H_c & F_c \end{bmatrix}$$

$$G = \begin{bmatrix} \psi_2(T) \\ 0 \\ G_c C_p \psi_2(\tau) \end{bmatrix}$$

where $\psi_1(\delta_c) = \psi_2(\delta_c) = 0$ and

$$H_1 = [E_c C_p \quad H_c \quad 0]$$

$$H_2 = [E_c C_p[\Phi(\tau) + \psi_1(\tau) E_c C_p] \quad E_c C_p \psi_1(\tau) H_c \quad H_c]$$

The equations of this case when δ_c is equal to zero are the same as the equations of the variation A or B when the time delay of these variations are equal to zero as expected.

4.3.2 Example

From the data of the first example of Section II

$A_p = 0$ $\quad F_c = 0$ $\quad W = \sigma_w^2$ then $w_k = \dfrac{\sigma_w^2}{T}$

$B_{1p} = 1$ $\quad G_c = 0$

$B_{2p} = 1$ $\quad H_c = 0$

$C_p = 1$ $\quad G_c = -k$

$$F = \begin{bmatrix} 1-k(T-\delta_c) & -k\delta_c & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_{xss} = \begin{bmatrix} \frac{\sigma_w{}^2 T}{1-(1-kT)^2} & 0 & 0 & \frac{\sigma_w{}^2 \tau}{1-(1-kT)(1-kT)} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$H_1 = [-k \quad 0 \quad 0 \quad 0 \quad 0]$$

$$H_2 = [-k \quad k^2\tau \quad 0 \quad 0 \quad 0]$$

Therefore,

$$P_{eAss} = \frac{k^2\tau^2\sigma_w{}^2}{T}$$

for $0 \leq \tau < \delta_c$.

and

$$P_{eBss} = \frac{k^4(T-\delta_c)^2\sigma_w{}^2 T}{kT(2-kT)} + \frac{k^2(T-\tau)^2\sigma_w{}^2}{T}$$

for $0 < \tau \leq \delta_c$.

P_{eAss} and P_{eBss} are plotted in Figure 20 as a function of τ. The diagrams to the right of each plot show the times corresponding to the values of the controller outputs used to calculate e_A and e_B. The same as variation A P_{eBss} depends on δ_c but P_{eBss} does not. The results obtained for this example agrees with intuition. For small τ, the variance of e_A is small and that of e_B is large. For large τ the complementary situation holds.

4.3.3 Variation C (Case II)

Because the input of the plant is the output of the first controller, therefore there isn't any effect in the system when the computation of y_{c2} isn't complete in the same period of the computation of y_{c1}. Therefore, the equations of this case are the same as the equations of Case I. The difference is the value of the sum of δ_c and τ, in this case $\delta_c+\tau<T$.

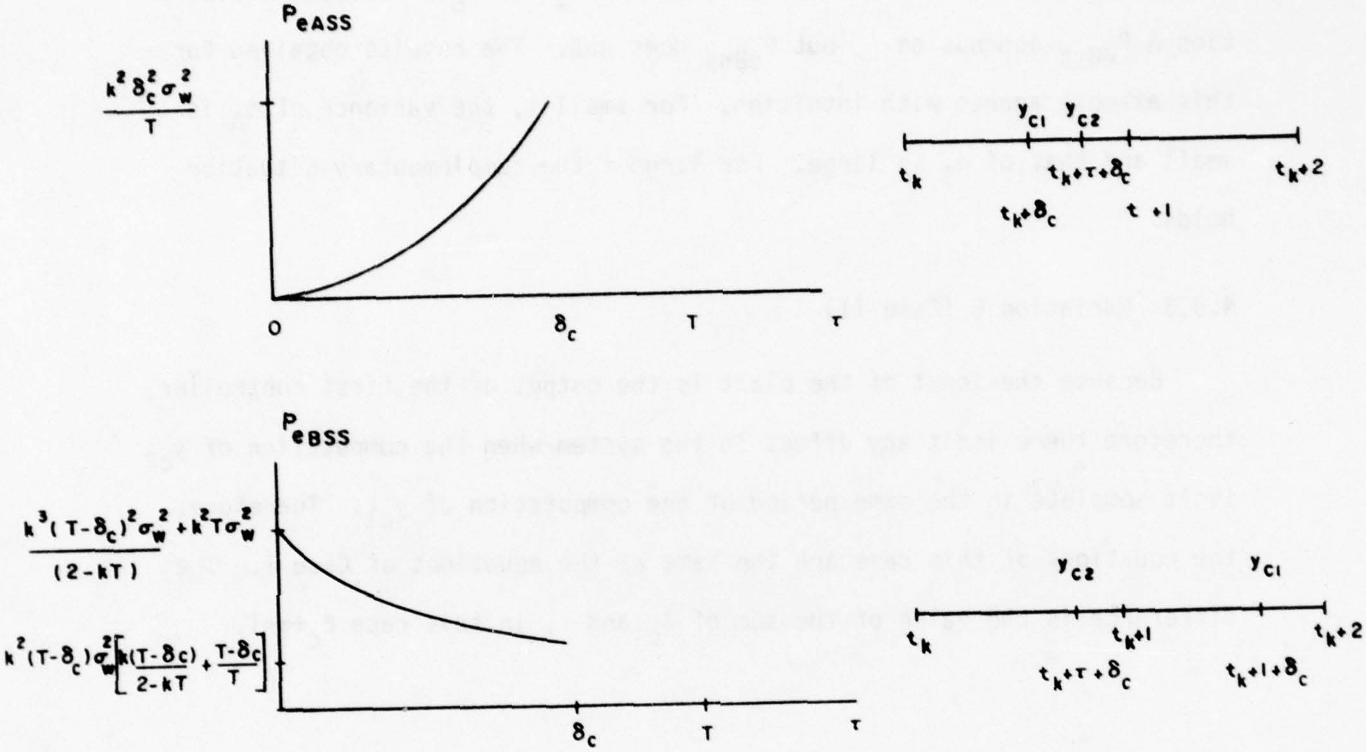


FIGURE 20 P_{eASS} and P_{eBSS} $0 < \tau < \delta_c$

5.0 THE OUTPUT-AVERAGING MODEL

This model is the same as the Delay Model except that the input of the plant is the average outputs of the controllers. There are three variations in this model, variation D, E and F. Variation D is nearly identical to the basic model (the value of the time delay is equal to zero). Variations E and F are nearly identical to variations B and C, respectively. Figure 21 illustrates the relationship between the plant and the controllers of this model.

5.1 Variation D: System Configuration and Dynamic Equations

Figure 22 shows the time responses of y_{c1} and y_{c2}. They are the same as the basic model, the aircraft, actuator and sensor dynamics. Therefore

$$x_p(t) = \Phi(t,t_o)x_p(t_o) + \psi_1(t,t_o)u_p(t_o) + \psi_2(t,t_o)w_p(t_o) \tag{5-1}$$

and

$$y_p(t) = C_p x_p(t) \tag{5-2}$$

In this model the input of the plant is the average output of the controllers.

$$u_p(t_k) = \frac{1}{2}\left[y_{c1}(t_k) + y_{c2}(t_k)\right] \tag{5-3}$$

$$u_{c1}(t_k) = y_p(t_k) = C_p x_p(t_k) \tag{5-4}$$

$$u_{c2}(t_k+1) = y_p(t_k+\tau) = C_p x_p(t_k+\tau) \tag{5-5}$$

The discrete-time equations for controller number 1 are:

$$x_{c1}(t_k+1) = F_c x_{c1}(t_k) + G_c u_{c1}(t_k) \tag{5-6}$$

$$y_{c1}(t_k) = H_c x_{c1}(t_k) + E_c U_{c1}(t_k) \tag{5-7}$$

for k = 0,1,..., and for controller number 2,

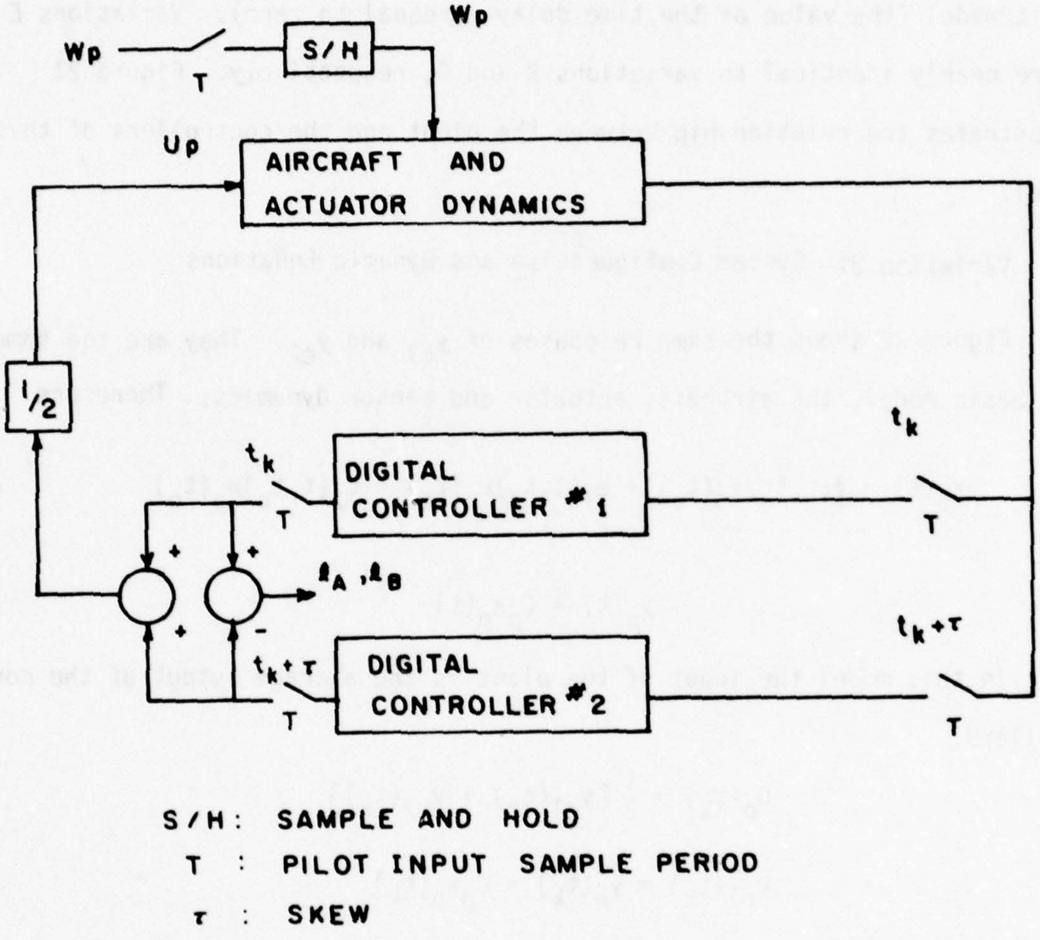


FIGURE 21 BLOCK DIAGRAM FOR THE OUTPUT-AVERAGE MODEL

FIGURE 22 THE TIME RESPONSES OF y_{C1} and y_{C2}

$$x_{c2}(t_k+1+\tau) = F_c x_{c2}(t_k+\tau) + G_c U_{c2}(t_k+\tau) \tag{5-8}$$

$$y_{c2}(t_k+\tau) = H_c x_{c2}(t_k+\tau) + E_c U_{c2}(t_k+\tau) \tag{5-9}$$

for $k = 0,1,\ldots$.

From Figure 22 $y_{c2}(t_k) = y_{c2}(t_k-1+\tau)$. Then by using (5-9)

$$y_{c2}(t_k) = y_{c2}(t_k-1+\tau) = H_c x_{c2}(t_k-1+\tau) + E_c U_{c2}(t_k-1+\tau) \tag{5-10}$$

Substituting (5-5) into (5-10) gives

$$y_{c2}(t_k) = y_{c2}(t_k-1+\tau) = H_c x_{c2}(t_k-1+\tau) + E_c C_p x_p(t_k-1+\tau) \tag{5-11}$$

Define new variables x_{hc2} and x_{hp1} in order to have variables that change in a piecewise constant manner. There are additional equations that are required, namely,

$$x_{hc2}(t_k+1+\tau) = x_{c2}(t_k+\tau) \tag{5-12}$$

and

$$x_{hp2}(t_k+1+\tau) = x_p(t_k+\tau) \tag{5-13}$$

Then (5-10) becomes

$$y_{c2}(t_k) = y_{c2}(t_k-1+\tau) - H_c x_{hc2}(t_k+\tau) + E_c C_p x_{hp2}(t_k+\tau) \tag{5-14}$$

Substituting (5-5) into (5-7) gives

$$y_{c1}(t_k) = H_c x_{c1}(t_k) + E_c C_p x_p(t_k) \tag{5-15}$$

Substituting (5-10) and (5-15) into (5-3) gives

$$u_p(t_k) = H_c x_{c1}(t_k) + E_c C_p x_p(t_k) + H_c x_{hc2}(t_k-\tau) + E_c C_p x_{hp2}(t_k+\tau) \tag{5-16}$$

At time $t_k+\tau$, the value of u_p changes and lets $t=t_k+\tau$ and $t_o=t_k$, (5-1) becomes

$$x_p(t_k+\tau) = \Phi(\tau)x_p(t_k) + \psi_1(\tau)u_p(t_k) + \psi_2(\tau)w_p(t_k) \qquad (5\text{-}17)$$

Substitution of equation (5-16) into (5-17) gives

$$x_p(t_k+\tau) = [\Phi(\tau) + \frac{\psi_1(\tau)}{2} E_cC_p]x_p(t_k) + \frac{\psi_1(\tau)}{2} H_cx_{c1}(t_k) + \frac{\psi_1(\tau)}{2} E_cC_px_{hp2}(t_k+\tau)$$

$$+ \frac{\psi_1(\tau)}{2} H_cx_{hc2}(t_k+\tau) + \psi_2(\tau)w_p(t_k) \qquad (5\text{-}18)$$

Now, let $t=t_{k+1}$ and $t_o=t_k+\tau$. Then (5-1) becomes

$$x_p(t_k+1) = \Phi(T-\tau)x_p(t_k+\tau) + \psi_1(T-\tau)u_p(t_k+\tau) + \psi_2(T-\tau)w_p(t_k) \qquad (5\text{-}19)$$

From Figure 22:

$$u_p(t_k+\tau) = \frac{1}{2}[y_{c1}(t_k) + y_{c2}(t_k+\tau)] \qquad (5\text{-}20)$$

Substituting (5-9) anu (5-15) with (5-20), gives

$$u_p(t_k+\tau) = \frac{1}{2}[H_cx_{c1}(t_k) + E_cC_px_p(t_k) + H_cx_{c2}(t_k+\tau) + E_cC_px_p(t_k+\tau)] \qquad (5\text{-}21)$$

Substitution of (5-11) into (5-19), gives

$$x_p(t_k+1) = [\Phi(T-\tau) + \frac{\psi_1(T-\tau)E}{2} E_cC_p]\, x_p(t_k+\tau) + \frac{\psi_1(T-\tau)}{2} H_cx_{c1}(t_k)$$

$$+ \frac{\psi_1(T-\tau)}{2} E_cC_px_p(t_k) + \frac{\psi_1(T-\tau)}{2} H_cx_{c2}(t_k+\tau) + \psi_2(T-\tau)w_p(t_k) \qquad (5\text{-}22)$$

Substitution of equations (5-18) with (5-22) gives

$$x_p(t_k+1) = \{[\Phi(T-\tau) + \psi_1\frac{(T-\tau)}{2} E_cC_p][\Phi(\tau) + \psi_1 \frac{(\tau)}{2} E_cC_p] + \psi_1 \frac{(T-\tau)}{2} E_cC_p\}x_p(t_k)$$

$$+ \{[\Phi(T-\tau) + \psi_1 \frac{(T-\tau)}{2} E_cC_p] \psi_1 \frac{(\tau)}{2} H_c + \psi_1 \frac{(T-\tau)}{2} H_c\} x_{c1}(t_k)$$

$$+ (\Phi(T-\tau) + \psi_1 \frac{(T-\tau)}{2} \psi_1 \frac{(\tau)}{2} E_cC_p \, x_{wp2}(t_k+\tau)$$

$$+ [\Phi(T-\tau) + \psi_1 \frac{(T-\tau)}{2} E_cC_p] \psi_1 \frac{(\tau)}{2} H_c \, x_{hc2}(t_k+\tau) + \psi_1 \frac{(T-\tau)}{2} H_c \, x_{c2}(t_k+\tau)$$

$$+ \{[\Phi(T-\tau) + \psi_1 \frac{(T-\tau)}{2} E_cC_p] \psi_2(\tau) + \psi_2(T-\tau)\} w_p(t_k) \tag{5-23}$$

Substituting (5-4) into (5-6) and substituting (5-5) into (5-8) gives

$$x_{c1}(t_k+1) = F_c \, x_{c1}(t_k) + G_cC_px_p(t_k) \tag{5-24}$$

and

$$x_{c2}(t_k+1+\tau) = F_cx_{c2}(t_k+\tau) + G_cC_px_p(t_k+1) \tag{5-25}$$

Substituting (5-18) into (5-25) gives

$$x_{c2}(t_k+1+\tau) = F_cx_{c2}(t_k) + G_cC_p[\Phi(\tau) + \psi_1 \frac{(\tau)}{2} E_cC_p]x_p(t_k) + G_cC_p\psi_1 \frac{(\tau)}{2}H_cx_{c1}(t_k)$$

$$+ G_cC_p \frac{\psi_1}{2} (\tau) \, E_cC_px_{hp2}(t_k+\tau) + G_cC_p \frac{\psi_1}{2} (\tau)H_cx_{hc2}(t_k+\tau)$$

$$+ G_cC_p \, {}_2(\tau)w_p(t_k) \tag{5-26}$$

The inherent error is defined in two parts as follows:

$$e_A(t) = y_{c1}(t_k) - y_{c2}(t_k+\tau) \tag{5-27}$$

for $t_k+\tau \le t \le t_k+1$, $k = 0,1,\ldots,$ $0 \le \tau \le T$, and

$$e_B(t) = y_{c1}(t_k+1) - y_{c2}(t_k+\tau) \tag{5-28}$$

for $t_k+1 \le t \le t_k+1+\tau$, $k = 0,1,\ldots,$ $0 \le \tau \le T$.

Figure 23 shows the skewed sampling and inherent errors of this case.

These equations can be put in compact from writing them in terms of a combined stated vector.

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{c1}(t_k) \\ x_{hp2}(t_k+\tau) \\ x_{c2}(t_k+\tau) \\ x_{hc2}(t_k+\tau) \end{bmatrix}, \quad x(t_k+1) = \begin{bmatrix} x_p(t_k+1) \\ x_{c1}(t_k+1) \\ x_{hp2}(t_k+1+\tau) \\ x_{c2}(t_k+1+\tau) \\ x_{hc2}(t_k+1+\tau) \end{bmatrix}$$

The state equations become

$$x(t_k+1) = F(T,\tau)\ x(t_k) + G(T,\tau)\ w_p(t_k) \tag{5-29}$$

where $F(T,\tau)$ is

$$f_{11} = [\Phi(T-\tau) + \psi_1\frac{(T-\tau)}{2}E_cC_p][\ (\tau) + \psi_1\ \frac{(\tau)}{2}E_cC_p] + \psi_1\frac{(T-\tau)}{2}E_cC_p$$

$$f_{12} = [\Phi(T-\tau) + \psi_1\frac{(T-\tau)}{2}E_cC_p]\ \psi_1\frac{(\tau)}{2}\ H_c + \psi_1\frac{(T-\tau)}{2}\ H_c$$

$$f_{13} = [\Phi(T-\tau) + \psi_1\frac{(T-\tau)}{2}E_cC_p]\ \psi_1\frac{(\tau)}{2}E_cC_p$$

$$f_{14} = \psi_1(T-\tau)\ H_c$$

$$f_{15} = [\Phi(T-\tau) + \psi_1\frac{(T-\tau)}{2}E_cC_p]\psi_1\frac{(\tau)}{2}\ H_c$$

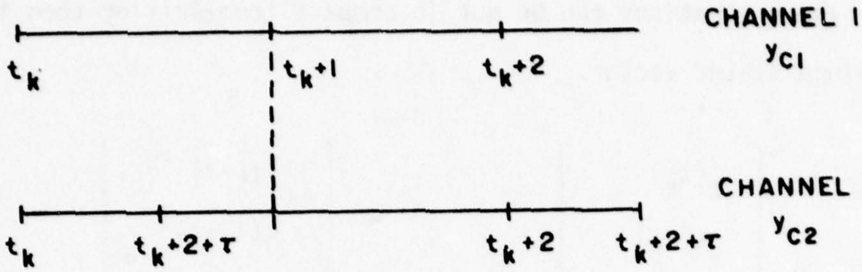
$$f_{21} = G_cC_p$$

$$f_{22} = F_c$$

$$f_{23} = f_{24} = f_{25} = 0$$

$$f_{31} = \Phi(\tau) + \psi_1\frac{(\tau)}{2}E_cC_p$$

$$f_{32} = \psi_1\frac{(\tau)}{2}E_cC_p$$



$$\ell_A(t) = y_{C1}(t_k) - y_{C2}(t_k+\tau)$$

FOR $\quad t_k+\tau \le t < t_k+1 \qquad k=0,1,\dots,0 \le \tau < T$

$$\ell_B(t) = y_{C1}(t_k+1) - y_{C2}(t_k+\tau)$$

FOR $\quad t_k+1 \le t < t_k+1+\tau \qquad k=0,1,\dots,0 < \tau \le T$

FIGURE 23 SKEWED SAMPLED AND INHERENT ERRORS

$$f_{34} = 0$$

$$f_{35} = \psi_1 \frac{(\tau)}{2} H_c$$

$$f_{41} = G_c C_p [\Phi(\tau) + \psi_1 \frac{(\tau)}{2} E_c C_p$$

$$f_{42} = G_c C_p \psi_1 \frac{(\tau)}{2} H_c$$

$$f_{43} = G_c C_p \psi_1 \frac{(\tau)}{2} E_c C_p$$

$$f_{44} = F_c$$

$$f_{45} = G_c C_p \psi_1 \frac{(\tau)}{2} H_c$$

$$f_{51} = f_{52} = f_{53} = 0$$

$$f_{54} = 1$$

$$f_{55} = 0$$

and $G(T,\tau)$ is

$$g_1 = [\Phi(T-\tau) + \psi_1 \frac{(T-\tau)}{2} E_c C_p] \psi_2(\tau) + \psi_2(T-\tau)$$

$$g_2 = 0$$

$$g_3 = \psi_2(\tau)$$

$$g_4 = G_c C_p \psi_2(\tau)$$

$$g_5 = 0$$

The controller output equations are

$$y_{c1}(t_k) = H_1 x(t_k) \tag{5-30}$$

and

$$y_{c2}(t_k+\tau) = H_2 x(t_k) + \rho \tag{5-31}$$

where

$$H_1 = [E_cC_p \quad H_c \quad 0 \quad 0 \quad 0] \tag{5-32}$$

$$H_2[E_cC_p[\Phi(\tau) + \psi_1\frac{(\tau)}{2}E_cC_p] \quad E_cC_p\psi_1\frac{(\tau)}{2}H_c \quad E_cC_p\psi_1\frac{(\tau)}{2}E_cC_p \quad H_c \quad E_cC_p\psi_1\frac{(\tau)}{2}H_c] \tag{5-33}$$

and

$$\rho = E_cC_p\,{}_2(\tau) \tag{5-34}$$

The expressions of $e_A(t)$ and $e_B(t)$ become

$$e_A(t) = (H_1-H_2)x(t_k) - \rho w_p(t_k) \tag{5-35}$$

for $t_k+\tau\leq t<t_k+1$, $k = 0,1,\ldots$, $0\leq\tau<T$, and

$$e_B(t) = (H_1F-H_2)x(t_k) + (H_1G-\rho)w_p(t_k) \tag{5-36}$$

for $t_k+1\leq t<t_k+1+\tau$, $k = 0,1,\ldots$, $0<\tau\leq T$.

5.1.1 Covariance Analysis

The analysis is the same as the delay model, then

$$P_k(k+1) = F(T,\tau)P_x(k)F^T(T,\tau) + G(T,\tau)w_kG^T(T,\tau) \tag{5-37}$$

The steady-state covariance is

$$P_{xss} = F(T,\tau)P_{xss}F^T(T,\tau) + G(T,\tau)w_kG^T(T,\tau) \tag{5-38}$$

where

$$P_x(k) = E[x(t_k)x^T(t_k)]$$

and

$$w_k = E[w_p(t_k)w_p{}^T(t_k)]$$

$$P_{eA}(t) = H_AP_x(k)H_A{}^T + \rho w_k\rho^T \tag{5-39}$$

for $t_k+\tau \le t < t_k+1$, $k = 0,1,\ldots$, $0 \le \tau < T$

where

$$H_A = H_1 - H_2 \tag{5-40}$$

and

$$P_{eB}(t) = H_B P_x(k) H_B^T + \rho_B w_k \rho_B^T \tag{5-41}$$

for $t_k+1 \le t < t_k+1+\tau$, $k = 0,1,\ldots$, $0 < \tau \le T$. Where

$$H_B = H_1 F - H_2 \tag{5-42}$$

and

$$\rho_B = H_1 G - \rho \tag{5-43}$$

5.1.2 Example

The data from the first example of Section II are:

$A_p = 0$ $\quad F_c = 0$

$B_{1p} = 1$ $\quad G_c = 0$

$B_{2p} = 1$ $\quad H_c = 0$

$C_p = 1$ $\quad E_c = -k$

$$w_k = \frac{\sigma_w^2}{T}$$

By using the equations of matrices $F(T,\tau)$ and $G(T,\tau)$ with these data, we have

$$F = \begin{bmatrix} [1-k\frac{(T-\tau)}{2}][1-\frac{kT}{2}] - k\frac{(T-\tau)}{2} & 0 & -[1-k\frac{(T-\tau)}{2}]\frac{kT}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1-\frac{kT}{2} & 0 & -\frac{kT}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{5-41}$$

$$G = \begin{bmatrix} [1-k(T-\tau)]t + (T-\tau) & = & T-kT(T-\tau) \\ 0 & & 0 \\ \tau & & \tau \\ 0 & & 0 \\ 0 & & 0 \end{bmatrix} \tag{5-45}$$

$$FP_{xss}F^T = \begin{bmatrix} [1-kT+k^2\tau\frac{(T-\tau)}{2}]^2 & 0 & [1-kT+k^2T\frac{(T-\tau)}{2}](1-kT) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ [1-kT+k^2T\frac{(T-\tau)}{2}](1-kT) & 0 & (1-kT)^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5-46}$$

and

$$Gw_kG^T = \begin{bmatrix} [T-k\tau(T-\tau)]^2 \frac{\sigma_w^2}{T} & 0 & [T-k\tau(T-\tau)] \frac{\tau\sigma_w^2}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ [T-k\tau(T-\tau)]\frac{\tau\sigma_w^2}{T} & 0 & \frac{\sigma_w^2\tau^2}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5-46}$$

Therefore by equation (5-38) we have

$$P_{xss} = \begin{bmatrix} \dfrac{[T-k\tau(T-\tau)]^2\sigma_w^{\;2}}{T[1-\{1-kT+k^2\tau\frac{(T-\tau)}{2}\}^2]} & 0 & \dfrac{[T-k\tau(T-\tau)]\tau\sigma_w^{\;2}}{T[1-\{1-kT+k^2\tau(T-\tau)\}\{1-kT\}]} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \dfrac{[T-k\tau(T-\tau)]\;\tau\sigma_w^{\;2}}{T[1-\{1-kT+k^2\tau\frac{(T-\tau)}{2}\}\{1-kT\}]} & 0 & \dfrac{\sigma_w^{\;2}\tau^2}{T[1-(1-kT)^2]} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From (5-32), (5-33), (5-34), (5-40), (5-42) and (5-43), we have

$$H_1 = [-k \quad 0 \quad 0 \quad 0 \quad 0] \tag{5-48}$$

$$H_2 = [-k(1-\frac{k\tau}{2}) \quad 0 \quad \frac{k^2\tau}{2} \quad 0 \quad 0] \tag{5-49}$$

$$H_A = [-\frac{k^2\tau}{2} \quad 0 \quad -\frac{k^2\tau}{2} \quad 0 \quad 0] \tag{5-50}$$

$$H_1F = [-k[1-\frac{k\tau}{2} - k(T-\tau) + k^2\tau\frac{(T-\tau)}{4}] \quad 0 \quad \frac{k^2T}{2}[1-k\frac{(T-\tau)}{2}] \quad 0 \quad 0] \tag{5-51}$$

$$H_B = [(k^2(T-\tau) - k^2\tau\,\frac{(T-\tau)}{4} \quad 0 \quad -\frac{k^3\tau}{4}(T-\tau) \quad 0 \quad 0] \tag{5-52}$$

$$H_1G = -k(T-k\tau(T-\tau)] \tag{5-53}$$

$$o_B = -k(T-\tau)\,[k\tau-1] \tag{5-54}$$

From (5-39) and (5-41) we have

$$P'_{eAss} = \frac{k^4\tau^2}{4}\,[\frac{[T-k\tau(T-\tau)]^2\,\sigma_w^{\;2}}{T[1-\{1-kT+k^2\tau\frac{(T-\tau)}{2}\}^2]} + \frac{2[T-kT(T-\tau)]\tau\sigma_w^{\;2}}{T[1-\{1-kT+\frac{k^2\tau}{2}(T-\tau)\}1-kT\;]} + \frac{\sigma_w^{\;2}T^2}{T[kT(2-kT)]}]$$

$$+\frac{k^2T^2\sigma_w^{\;2}}{T} \tag{5-55}$$

and

$$
\begin{aligned}
P'_{eBss} &= [k^2(T-\tau)-k^3T\,\frac{(T-\tau)}{4}]^2[\frac{[T-kT(T-\tau)]^2\sigma_w^{\,2}}{T[1-\{1-kT+k^2T\frac{(T-\tau)}{2}]^2]}] \\
&- 2[k^2(T-\tau)-k^3\tau\frac{(T-\tau)}{4}]\;k^3T(T-\tau)[\frac{[T-kT(T-\tau)]\;\tau\sigma_w^{\,2}}{T[1-\{1-kT+k^2\tau(T-\tau)\}(1-kT)} \\
&+ k^2T^2\,\frac{(T-\tau)^2}{16}\,[\frac{\sigma_w^{\,2}\tau^2}{T[kT(2-kT)]}] + k^2\,\frac{(T-\tau)^2[kT-1]^2\sigma_w^{\,2}}{T}
\end{aligned}
\tag{5-56}
$$

P'_{eAss} and P'_{eBss} are plotted in figure 24 is a function of τ. The diagrams to the right of each plot show the times corresponding to the value of the controller output used to calculate e_A and e_B. The results obtained for this example agree with intuition. For small τ, the variance of e_A is small and that of e_B is large. For large τ the complimentary situation holds.

5.2 Variation E: System Configuration and Dynamic Equations

Figure 25 shows the time responses of y_{c1} and y_{c2}. They are the same as the basic model, the aircraft, actuator and sensor dynamics. Therefore,

$$x_p(t) = \Phi(t,t_o)x_p(t_o) + \psi_1(t,t_o)u_p(t_o) + \psi_2(t,t_o)w_p(t_o) \tag{5-57}$$

and

$$y_p(t) = C_p x_p(t) \tag{5-58}$$

In this variation, the input of the plant is the average output of the controllers and the output of the plant is the input to each controller at the different times. Then

$$U_p(t_k) = \frac{1}{2}[y_{c1}(t_k) + y_{c2}(t_k)] \tag{5-59}$$

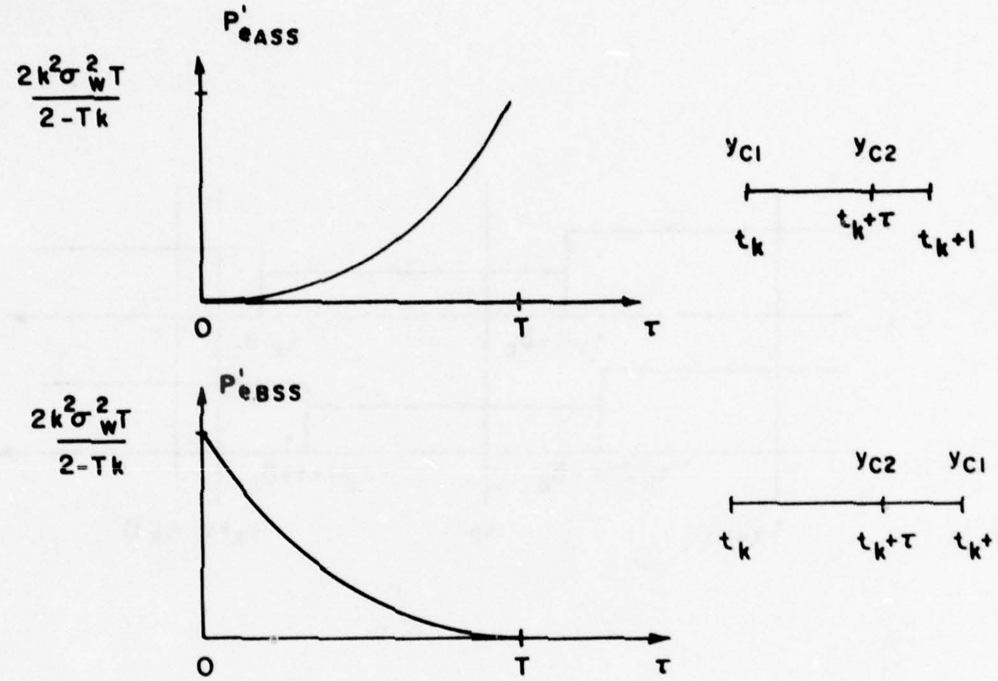


FIGURE 24 P'_{eASS} and P'_{eBSS} $0 < \tau < T$

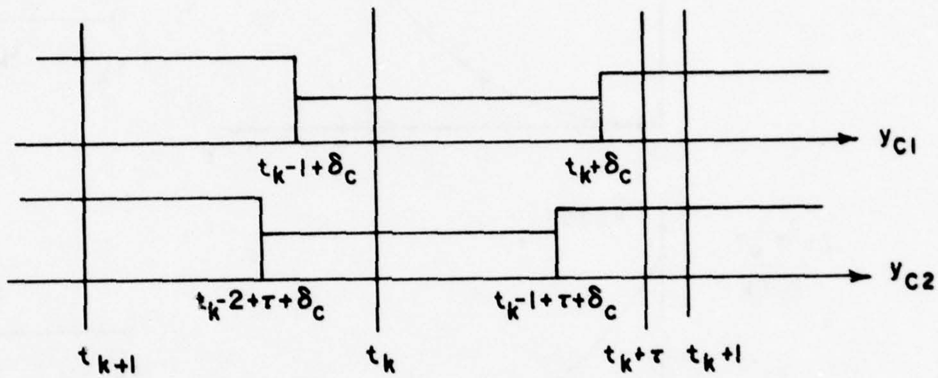


FIGURE 25 THE TIME RESPONSES OF y_{C1} and y_{C2}

$$U_{c1}(t_k) = y_p(t_k) = C_p x_p(t_k) \tag{5-60}$$

$$U_{c2}(t_k+\tau) = y_p(t_k+\tau) = C_p x_p(t_k+\tau) \tag{5-61}$$

The discrete-time equations for controller number one are:

$$x_{c1}(t_k+1+\delta_c) = F_c x_{c1}(t_k+\delta_c) + G_c U_{c1}(t_k) \tag{5-62}$$

$$y_{c1}(t_k+\delta_c) = H_c x_{c1}(t_k+\delta_c) + E_c U_{c1}(t_k) \tag{5-63}$$

for k = 0,1,..., and for controller number two,

$$x_{c2}(t_k+1+\tau+\delta_c) = F_c x_{c2}(t_k+\tau+\delta_c) + G_c U_{c2}(t_k+\tau) \tag{5-64}$$

$$y_{c2}(t_k+\tau+\delta_c) = H_c x_{c2}(t_k+\tau+\delta_c) + E_c U_{c2}(t_k+\tau) \tag{5-65}$$

for k = 0,1,... .

From figure 25 $y_{c2}(t_k) = y_{c2}(t_k-2+\tau+\delta_c)$ then by using (5-65)

$$y_{c2}(t_k) = y_{c2}(t_k-2+\tau+\delta_c) = H_c x_{c2}(t_k-2+\tau+\delta_c) + E_c U_{c2}(t_k-2+\tau) \tag{5-66}$$

Substituting (5-61), with (5-66) gives

$$y_{c2}(t_k) = y_{c2}(t_k-2+\tau+\delta_c) = H_c x_{c2}(t_k-2+\tau+\delta_c) + E_c C_p x_p(t_k-2+\tau) \tag{5-67}$$

From figure 25 $y_{c1}(t_k) = y_{c1}(t_k-1+\delta_c)$ by using (5-63)

$$y_{c1}(t_k) = y_{c1}(t_k-1+\delta_c) = H_c x_{c1}(t_k-1+\delta_c) + E_c C_p x_p(t_k-1) \tag{5-68}$$

Define the new variables x_{hc1}, x_{hp1}, x_{hhc2}, x_{hc2}, x_{hhp2} and x_{hp2} in order to have the variables that change in a piecewise constant manner. There are additions that are required, namely,

$$x_{hc1}(t_k+1+\delta_c) = x_{c1}(t_k+\delta_c) \tag{5-69}$$

$$x_{hp1}(t_k+1) = x_p(t_k) \tag{5-70}$$

$$x_{hhc2}(t_k+1+\tau+\delta_c) = x_{hc2}(t_k+\tau+\delta_c) \tag{5-71}$$

$$x_{hc2}(t_k+1+\tau+\delta_c) = x_{c2}(t_k+\tau+\delta_c) \tag{5-72}$$

$$x_{hhp2}(t_k+1+\tau) = x_{hp2}(t_k+\tau) \tag{5-73}$$

and

$$x_{hp2}(t_k+1+\tau) = x_p(t_k+\tau) \tag{5-74}$$

By using (5-69), (5-70), (5-71), (5-72), (5-73) and (5-74), (5-67) and (5-68) become

$$y_{c2}(t_k) - y_{c2}(t_k-2+\tau+\delta_c) = H_c x_{hhc2}(t_k+\tau+\delta_c) + E_c C_p x_{hhp2}(t_k+\tau) \tag{5-75}$$

$$y_{c1}(t_k) - y_{c1}(t_k-1+\delta_c) = H_c x_{hc1}(t_k+\delta_c) + E_c C_p x_{hp1}(t_k) \tag{5-76}$$

Substituting (5-75) and (5-76) into (5-59) gives,

$$U_p(t_k) = \tfrac{1}{2}[H_c x_{hc1}(t_k+\delta_c) + E_c C_p x_{hp1}(t_k) + H_c x_{hhc2}(t_k+\tau+\delta_c) + E_c C_p x_{hhp2}(t_k+\tau)] \tag{5-77}$$

At time $t_k-1+\tau+\delta_c$ the value of U_p changes, so let $t=t_k-1+\tau+\delta_c$ and $t_o=t_k$. Then (5-57) becomes

$$x_p(t_k-1+\tau+\delta_c) = \Phi(\tau+\delta_c-T)x_p(t_k) + \psi_1(\tau+\delta_c-T)U_p(t_k) + \psi_2(\tau+\delta_c-T)w_p(t_k) \tag{5-78}$$

Substituting (5-77) into (5-78) gives

$$x_p(t_k-1+\tau+\delta_c) = \Phi(\tau+\delta_c-T)x_p(t_k) + \psi_1\frac{(\tau+\delta_c-T)}{2}H_c x_{hc1}(t_k+\delta_c)$$

$$+ \psi_1\frac{(\tau+\delta_c-T)}{2}E_c C_p x_{hp1}(t_k)w_p(t_k) + \psi_1\frac{(\tau+\delta_c-T)}{2}H_c x_{hhc2}(t_k+\tau+\delta_c)$$

$$+ \psi_1\frac{(\tau+\delta_c-T)}{2}E_c C_p x_{hhp2}(t_k+\tau) + \psi_2(\tau+\delta_c-T) \tag{5-79}$$

At time $t_k+\delta_c$ the value of U_p changes again so let $t=t_k+\delta_c$ and $t_o=t_k-1+\tau+\delta_c$ then

$$x_p(t_k+\delta_c) = \Phi(T-\tau)x_p(t_k-1+\tau+\delta_c) + \psi_1(T-\tau)U_p(t_k-1+\tau+\delta_c) + \psi_2(T-\tau)w_p(t_k) \tag{5-80}$$

Consider $U_p(t_k-1+\tau+\delta_c)$. From figure 25

$$U_p(t_k-1+\tau+\delta_c) = \frac{1}{2}[y_{c1}(t_k-1+\tau+\delta_c) + y_{c2}(t_k-1+\tau+\delta_c)] = \frac{1}{2}[y_{c1}(t_k-1+\delta_c)$$

$$+ y_{c2}(t_k-1+\tau+\delta_c)] \tag{5-81}$$

By using (5-61) and (5-65)

$$y_{c2}(t_k-1+\tau+\delta_c) = H_c x_{c2}(t_k-1+\tau+\delta_c) + E_c C_p x_p(t_k-1+\tau) \tag{5-82}$$

By using (5-72) and (5-74), becomes

$$y_{c2}(t_k-1+\tau+\delta_c) - H_c x_{hc2}(t_k+\tau+\delta_c) + E_c C_p x_{hp2}(t_k+\tau) \tag{5-83}$$

Substituting (5-76) and (5-83) into (5-81) gives

$$U_p(t_k-1+\tau+\delta_c) = \frac{1}{2}[H_c x_{hc1}(t_k+\delta_c) + E_c C_p x_{hp1}(t_k) + H_c x_{hc2}(t_k+\tau+\delta_c)$$

$$+ E_c C_p x_{hp2}(t_k+\tau)] \tag{5-84}$$

Substituting (5-79) and (5-84) into (5-80) gives

$$x_p(t_k+\delta_c) = \Phi(\delta_c)x_p(t_k) + [\Phi(T-\tau)\psi_1(\frac{\tau+\delta_c-T}{2})H_c + \psi_1(\frac{T-\tau}{2})H_c](t_k+\delta_c)$$

$$+ [\Phi(T-\tau)\psi_1(\frac{\tau+\delta_c-T}{2})E_cC_p + \psi_1\frac{(T-\tau)}{2}E_cC_p]x_{hp1} +$$

$$+ \Phi(T-\tau)\frac{\psi_1(\tau+\delta_c-T)}{2}H_cx_{hhc2}(t_k+\tau+\delta_c) + \Phi(T-\tau)\frac{\psi_1(\tau+\delta_c-T)}{2}E_cC_px_{hhp2}(t_k+\tau)$$

$$+ \psi_1(T-\tau)H_cx_{hc2}(t_k+\tau+\delta_c) + \psi_1\frac{(T-\tau)}{2}E_cC_px_{hp2}(t_k+\tau)$$

$$+ [\Phi(T-\tau)\psi_2(\tau+\delta_c-T) + \psi_2(T-\tau)]\, w_p(t_k) \tag{5-85}$$

Now let $t=t_k+1$ and $t_o=t_k+\delta_c$. Then (5-57) becomes

$$x_p(t_k+1) = \Phi(T-\delta_c)x_p(t_k+\delta_c) + \psi_1(T-\delta_c)U_p(t_k+\delta_c) + \psi_2(T-\delta_c)w_p(t_k) \tag{5-86}$$

Consider $U_p(t_k+\delta_c)$

From Figure 25

$$U_p(t_k+\delta_c) = \tfrac{1}{2}[y_{c1}(t_k+\delta_c) + y_{c2}(t_k+\delta_c)] = \tfrac{1}{2}[y_{c1}(t_k+\delta_c)$$

$$+ y_{c2}(t_k-1+\tau+\delta_c)] \tag{5-87}$$

By using (5-58) and (5-63)

$$y_{c1}(t_k+\delta_c) = H_cx_{c1}(t_k+\delta_c) + E_cC_px_p(t_k) \tag{5-88}$$

Substituting (5-88) and (5-83) into (5-87) gives

$$U_p(t_k+\delta_c) = \tfrac{1}{2}[H_cx_{c1}(t_k+\delta_c) + E_cC_px_p(t_k) + H_cx_{hc2}(t_k+\tau+\delta_c) + E_cC_px_{hp2}(t_k+\tau) \tag{5-89}$$

Substituting (5-85) and (5-89) into (5-86) gives

$$x_p(t_k+1) = [\Phi(T) + \psi_1\frac{(T-\delta_c)}{2} E_cC_p]x_p(t_k) + \Phi(T-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} H_c$$

$$+ \psi_1\frac{(T-\tau)}{2} H_c]x_{hc1}(t_k+\delta_c) + \Phi(T-\delta_c)[\Phi(T-\tau)\frac{\psi_1(\tau+\delta_c-T)}{2} E_cC_p +$$

$$+ \psi_1\frac{(T-\tau)}{2} E_cC_p]x_{hp1}(t_k) + \Phi(T-\delta_c)\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} H_cx_{hhc2}(t_k+\tau+\delta_c)$$

$$+ \Phi(T-\delta_c)\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} E_cC_p\, x_{hhp2}(t_k+\tau) + [\Phi(T-\delta_c)\psi_1\frac{(T-\tau)}{2} H_c$$

$$+ \psi_1\frac{(T-\delta_c)}{2} H_c]x_{hc2}(t_k+\tau+\delta_c) + [\ \Phi(T-\delta_c)\psi_1\frac{(T-\tau)}{2} E_cC_p$$

$$+ \psi_1\frac{(T-\delta_c)}{2} E_cC_p]x_{hhp2}(t_k+\tau) + \psi_1\frac{(T-\delta_c)}{2} H_cx_{c1}(t_k+\delta_c)[\Phi(T-\tau)\psi_2(\tau+\delta_c-T)$$

$$+ \psi_2(T-\tau)] + \psi_2(T-\delta_c)x_{hp2}(t_k+\tau)w_p(t_k) \tag{5-90}$$

Substituting (5-58) and (5-60) into (5-62) and substituting (5-58) and (5-61) into (5-64) gives,

$$x_{c1}(t_k+1+\delta_c) = F_cx_{c1}(t_k+\delta_c) + G_cC_px_p(t_k) \tag{5-91}$$

and

$$x_{c2}(t_k+1+\tau+\delta_c) = F_cx_c(t_k+\tau+\delta_c) + G_cC_px_p(t_k+\tau) \tag{5-93}$$

In this case the quantity $x_p(t_k+\tau)$ can be written using the solution to equation (5-91):

$$x_p(t_k+\tau) = [\Phi(\tau) + \psi_1\frac{(\tau-\delta_c)}{2} E_cC_p]x_p(t_k) + \Phi(\tau-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} H_c$$

$$+ \psi_1\frac{(T-\tau)}{2} H_c]x_{hc1}(t_k) + \Phi(\tau-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} E_cC_p + \psi_1\frac{(T-\tau)}{2} E_cC_p]x_{hp1}(t_k)$$

$$+ \Phi(T-\delta_c)\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} H_cx_{hhc2}(t_k+\tau+\delta_c) + \Phi(\tau-\delta_c)\ \frac{(T-\tau)}{2} E_cC_p$$

$$+ \psi_1 \frac{(\tau-\delta_c)}{2} E_c C_p] x_{hp2}(t_k+\tau) + \psi_1 \frac{(\tau-\delta_c)}{2} H_c x_{c1}(t_k+\delta_c) + \{\Phi(T-\tau)\psi_2(\tau+\delta_c-T)$$

$$+ \psi_2(T-\tau)] + \psi_2(\tau-\delta_c)\} \, w_p(t_k) \tag{5-92}$$

The inherent error is defined in two parts as follows:

$$e_A(t) - y_{c1}(t_k+\delta_c) - y_{c2}(t_k+\tau+\delta_c) \tag{5-93}$$

for $t_k+\tau+\delta_c \leq t < t_k+\tau+\delta_c$, $k = 0,1,\ldots,$ $\delta_c \leq \tau < T$

and

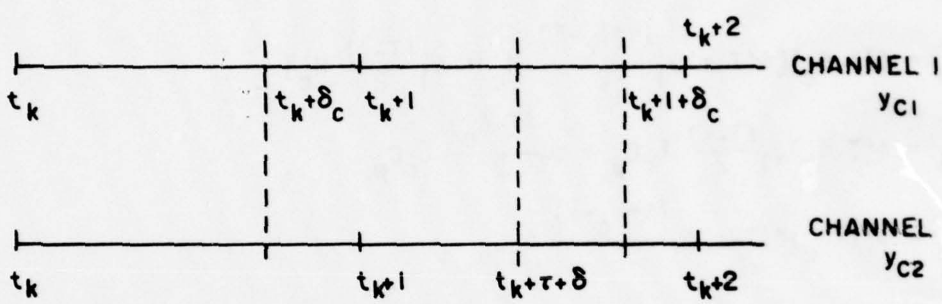
$$e_B(t) = y_{c1}(t_k+1+\delta_c) - y_{c2}(t_k+\tau+\delta_c) \tag{5-94}$$

for $t_k+1+\delta_c \leq t < t_k+1+\tau+\delta_c$, $k = 0,1,\ldots,$ $\delta_c<\tau\leq T$.

Figure 26 shows the skewed sampling and inherent errors. These equations can be put in compact form by writing them in terms of a combined stated vector

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{hp1}(t_k) \\ x_{c1}(t_k+\delta_c) \\ x_{hc1}(t_k+\delta_c) \\ x_{hp2}(t_k+\tau) \\ x_{hhp2}(t_k+\tau) \\ x_{c2}(t_k+\tau+\delta_c) \\ x_{hc2}(t_k+\tau+\delta_c) \\ x_{hhc2}(t_k+\tau+\delta_c) \end{bmatrix}, \quad x(t_k+1) = \begin{bmatrix} x_p(t_k+1) \\ x_{hp1}(t_k+1) \\ x_{c1}(t_k+1+\delta_c) \\ x_{hc1}(t_k+1+\tau) \\ x_{hp2}(t_k+1+\tau) \\ x_{hhp2}(t_k+1+\tau) \\ x_{c2}(t_k+1+\tau+\delta_c) \\ x_{hc2}(t_k+1+\tau+\delta_c) \\ x_{hhc2}(t_k+1+\tau+\delta_c) \end{bmatrix}$$

The states equations become



$\ell_A(t) = y_{C1}(t_k+\delta_c) - y_{C2}(t_k+\tau+\delta_c)$

FOR $t_k+\tau+\delta_c \le t < t_k+1+\delta_c$, $k=0,1,\ldots,\ \delta \le \tau < T$

$\ell_B(t) = y_{C1}(t_k+1+\delta_c) - y_{C2}(t_k+\quad+\delta_c)$

FOR $t_k+1+\delta_c \le t < t_k+1+\tau+\delta_c$ $k=0,1,\ldots,\ \delta_c < \tau \le T$

FIGURE 26 SKEWED SAMPLING AND INHERENT ERRORS

$$x(t_k+1) = F(t,\tau)x(t_k) + G(T,\tau)w_p(t_k) \tag{5-95}$$

where $F(T,\tau)$ is

$$f_{11} = \Phi(T) + \psi_1\frac{(T-\delta_c)}{2} E_cC_p]$$

$$f_{12} = \Phi(T-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} E_cC_p + \psi_1\frac{(T-\tau)}{2} E_cC_p]$$

$$f_{13} = \psi_1\frac{(T-\delta_c)}{2} H_c$$

$$f_{14} = \Phi(T-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} H_c + \psi_1\frac{(T-\tau)}{2} H_c]$$

$$f_{15} = \Phi(T-\delta_c)\psi_1\frac{(T-\tau)}{2} E_cC_p + \psi_1\frac{(T-\delta_c)}{2} E_cC_p$$

$$f_{16} = \Phi(T-\delta_c)\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} E_cC_p$$

$$f_{17} = 0$$

$$f_{18} = \Phi(T-\delta_c)\psi_1\frac{(T-\tau)}{2} H_c + \psi_1\frac{(T-\delta_c)}{2} H_c$$

$$f_{19} = \Phi(T-\delta_c)\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} H_c$$

$$f_{21} = 1,\ f_{22} = f_{23} = f_{24} = f_{25} = f_{26} = f_{27} = f_{28} = f_{29} = 0$$

$$f_{31} = G_cC_p$$

$$f_{32} = 0$$

$$f_{33} = F_c$$

$$f_{34} = f_{35} = f_{36} = f_{37} = f_{38} = f_{39} = 0$$

$$f_{41} = 0 = f_{42}$$

$$f_{43} = 1$$

$$f_{44} = f_{45} = f_{46} = f_{47} = f_{48} = f_{49} = 0$$

$$f_{51} = \Phi(\tau) + \psi_1\frac{(\tau-\delta_c)}{2}\,E_cC_p$$

$$f_{52} = \Phi(\tau-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}\,E_cC_p + \psi_1\frac{(T-\tau)}{2}\,E_cC_p$$

$$f_{53} = \psi_1\frac{(\tau-\delta_c)}{2}\,H_c$$

$$f_{54} = \Phi(\tau-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}\,H_c + \psi_1\frac{(T-\tau)}{2}\,H_c$$

$$f_{55} = \Phi(\tau-\delta_c)\psi_1\frac{(T-\tau)}{2}\,E_cC_p + \psi_1\frac{(\tau-\delta_c)}{2}\,E_cC_p$$

$$f_{56} = \Phi(\tau-\delta_c)\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}\,E_cC_p$$

$$f_{57} = 0$$

$$f_{58} = \Phi(\tau-\delta_c)\psi_1\frac{(T-\tau)}{2}\,H_c + \psi_1\frac{(\tau-\delta_c)}{2}\,H_c$$

$$f_{59} = \Phi(\tau-\delta_c)\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}\,H_c$$

$$f_{61} = f_{62} = f_{63} = f_{64} = 0$$

$$f_{65} = 1$$

$$f_{66} = f_{67} = f_{28} = f_{69} = 0$$

$$f_{71} = G_cC_p\,f_{51}$$

$$f_{72} = G_cG_p\,f_{c2}$$

$$f_{73} = G_cC_p\,f_{c3}$$

$$f_{74} = G_cC_p\,f_{54}$$

$$f_{75} = G_cC_p\,f_{55}$$

$$f_{76} = G_cC_p\,f_{56}$$

$$f_{77} = F_c$$

$$f_{78} = G_cC_p\,f_{58}$$

$$f_{79} = G_cC_p\,f_{59}$$

$f_{81} = f_{82} = f_{83} = f_{84} = f_{85} = f_{86}$

$f_{87} = 1$

$f_{88} = f_{89} = 0$

$f_{91} = f_{92} = f_{93} = f_{94} = f_{95} = f_{96} = f_{97} = 0$

$f_{98} = 1$

$f_{99} = 0$

and $G(T,\tau)$ is

$g_1 = \Phi(T-\delta_c)[\Phi(T-\tau)\psi_2(\tau+\delta_c-T) + \psi_2(T-\tau)] + \psi_2(T-\delta_c)$

$g_2 = 0 = g_3 = g_4$

$g_5 = \Phi(T-\delta_c)[\Phi(T-\tau)\psi_2(\tau+\delta_c-T) + \psi_2(T-\tau)] + \psi_2(\tau-\delta_c)$

$g_6 = 0$

$g_7 = G_cC_p\, g_5$

$g_8 = 0 = g_9$

The controller output equations are

$$y_{c1}(t_k+\delta_c) = H_1x(t_k) \tag{5-96}$$

and

$$y_{c2}(t_k+\tau+\delta_c) = H_2x(t_k) + \rho \tag{5-97}$$

then

$$H_1 = [E_cC_p \quad 0 \quad H_c \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{5-98}$$

$$H_2 = [E_cC_pf_{51} \quad E_cC_pf_{52} \quad E_cC_pf_{53} \quad E_cC_pf_{54} \quad E_cC_pf_{55} \quad E_cC_pf_{56} \quad H_c \quad E_cC_pf_{58} \quad E_cC_pf_{59}] \tag{5-99}$$

and

$$\rho = E_cC_pg_5 \tag{5-100}$$

The equations of $e_A(t)$ and $e_B(t)$ become

$$e_A(t) = (H_1-H_2)x(t_k) = \rho w_p(t_k) \tag{5-101}$$

for $t_k+\tau+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,\ldots,$ $\delta_c \leq \tau < T$

and

$$e_B(t) = (H_1F-H_2)x(t_k) + (H_1G-\rho)w_p(t_k) \tag{5-102}$$

for $t_k+1+\delta_c \leq t < t_k+1+\tau+\delta_c$, $\delta_c < \tau \leq T$.

5.2.1 Covariance Analysis

The analysis is the same as Variation D. In this variation of the value of time delay (δ_c) is equal to zero. Then this variation is the same as variation D. Therefore, if the time delay (δ_c) is equal to zero, the errors of this variation must be equal to the covariance errors of variation D.

5.2.2 Example

With data from the first example in Section II. Section $F(T,\tau)$ and $G(T,\tau)$ are:

$$F = \begin{bmatrix} 1-k\frac{(T-\delta_c)}{2} & \frac{-k\delta_c}{2} & 0 & 0 & \frac{-k}{2}(2T-\tau-\delta_c) & \frac{-k}{2}(\tau+\delta_c-T) & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1-k\frac{(\tau-\delta_c)}{2} & \frac{-k\delta_c}{2} & 0 & 0 & 0 & \frac{-k}{2}[T-\delta_c] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} T \\ 0 \\ 0 \\ 0 \\ \tau \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By using equations of P_{xss}

$$P_{xss} = \begin{bmatrix} \frac{\sigma_w{}^2 T}{kT(2-kT)} & 0 & 0 & 0 & \frac{\sigma_w{}^2 T}{1-(1-kT)(1-kT)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sigma_w{}^2 T}{1-(1-kT)(1-kT)} & 0 & 0 & 0 & \frac{\sigma_w{}^2 \tau^2}{kT\tau(2-k\tau)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5\text{-}104)$$

By using equations (5-98), (5-99), (5-100) and equations of H_A, H_B and σ_B

$$H_1 = [-k \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \quad (5\text{-}105)$$

$$H_2 = [-k(1-k\frac{(\tau-\delta_c)}{2}] \quad \frac{k^2}{2}\delta_c \quad 0 \quad 0 \quad k^2\frac{(T-\tau)}{2} \quad k^2\,\frac{(T+\delta_c-T)}{2} \quad 0 \quad 0 \quad 0] \quad (5\text{-}106)$$

$$H_A = [-k^2 \frac{(\tau-\delta_c)}{2} - \frac{k^2}{2}\delta_c \quad 0 \quad 0 \quad -k^2 \frac{(T-\delta_c)}{2} \quad -k^2 \frac{(\tau+\delta_c-T)}{2} \quad 0 \quad 0 \quad 0] \tag{5-107}$$

$$\rho = -kT$$

$$H_1F = [-k[1-k \frac{(T-\delta_c)}{2}] \quad \frac{k^2\delta_c}{2} \quad 0 \quad 0 \quad \frac{k^2}{2}(2T-\tau-\delta_c) \quad \frac{k^2}{2}(\tau+\delta_c-T) \quad 0 \quad 0 \quad 0]$$

$$H_B = [\frac{k^2}{2}(T-\tau) \quad \frac{-k^2\delta_c}{2} \quad 0 \quad 0 \quad \frac{-k^2}{2}(T-\tau) \quad 0 \quad 0 \quad 0 \quad 0]$$

$$H_1G = -kT$$

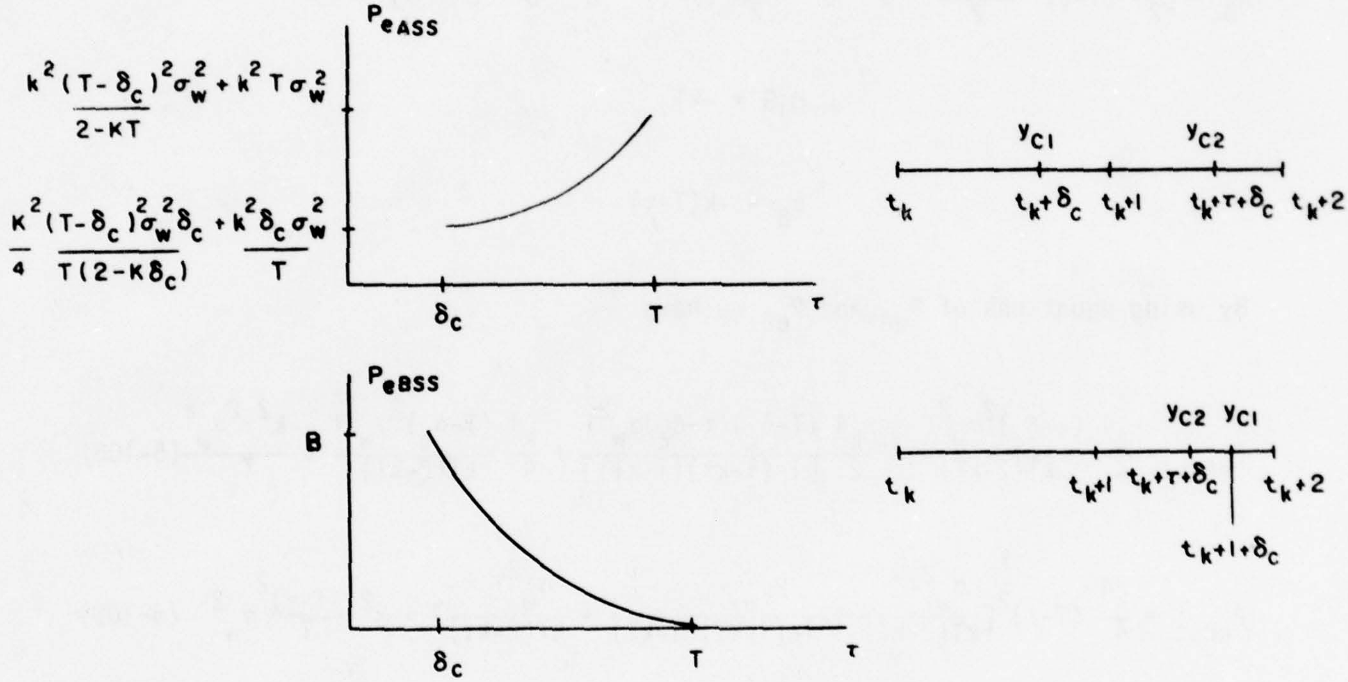
$$\rho_B = -k(T-\tau)$$

By using equations of P_{eA} and P_{eB} we have

$$P_{eAss} = \frac{k^4}{4}\frac{(\tau-\delta_c)^2\sigma_w{}^2T}{kT(2-kT)} + \frac{k^4}{2}\frac{(T-\delta_c)(\tau-\delta_c)\sigma_w{}^2T}{[1-(1-k\tau)(1-kT)]} + \frac{k^4}{4}\frac{(T-\delta_c)^2\sigma_w{}^2T}{kT(2-kT)} + \frac{k^2\tau^2\sigma_w{}^2}{T} \tag{5-108}$$

$$P_{eBss} = \frac{k^4}{4}(T-\tau)^2[\frac{\sigma_w{}^2T}{kT(2-kT)} + \frac{2\sigma_w{}^2\tau}{1-(1-kT)(1-kT)} + \frac{\sigma_w{}^2T}{kT(2-k\tau)}] + k^2\frac{(T-\tau)^2}{T}\sigma_w{}^2 \tag{5-109}$$

The same as the delay model, P_{eAss} depends on time delay δ_c but P_{eBss} doesn't. P_{eAss} *and* P_{eBss} are plotted in Figure 27 as a function of τ. The diagrams to the right of each plot show the times corresponding to the values of the controller outputs used to calculate e_A and e_B. The results obtained for this example agree with intuition. For small τ, the variance of e_A is small and that of e_B is large. For large τ the complimentary situation holds.



$$\text{WHERE } B = \frac{k^2\sigma_w^2}{4}(T-\delta_c)^2\left[\frac{k}{2-kT} + \frac{2k^2\delta_c}{1(1-kT)(1-k\delta_c)} + \frac{k\delta_c}{T(2-k\delta_c)} + \frac{4}{T}\right]$$

FIGURE 27 P_{eASS} and P_{eBSS} $\delta_c < \tau < T$

5.3 Variation F

The same as variation C, there are two cases in this variation.

1. The computation of y_{c2} is completed in the same period of the computation of $y_{c1}(\tau+\delta_c<T)$.
2. The computationof y_{c2} is not completed in the same period of the computation of $y_{c2}(\tau+\delta_c<T)$.

5.3.1 Variation F (Case I): System Configuration and Dynamic Equations

Figure 28 shows the time-responses of y_{c1} and y_{c2}. The same as the basic model, for the aircraft, actuator, and sensor dynamics. Therefore:

$$x_p(t) = \Phi(t,t_o)x_p(t_o) + \psi_1(t,t_o)U_p(t_o) + \psi_2(t,t_o)w_p(t_o) \tag{5-112}$$

and

$$y_p(t) = C_p x_p(t) \tag{5-113}$$

In this model, the input of the plant is the average outputs of the controllers so

$$U_p(t_k) = \frac{1}{2}[y_{c1}(t_k) + y_{c2}(t_k) \tag{5-114}$$

$$u_{c1}(t_k) = y_p(t_k) = C_p x_p(t_k) \tag{5-115}$$

$$u_{c2}(t_k+\tau) = y_p(t_k+\tau) = C_p x_p(t_k+\tau) \tag{5-116}$$

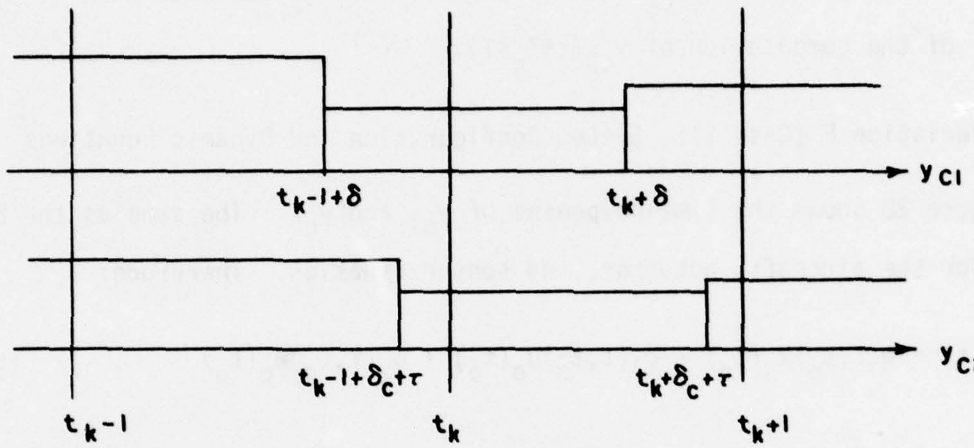
The discrete-time equations for controller number one are:

$$x_{c1}(t_k+1+\delta_c) = F_c x_{c1}(t_k+\delta_c) + G_c C_p x_p(t_k) \tag{5-117}$$

and

$$y_{c1}(t_k+\delta_c) = H_c x_{c1}(t_k+\delta_c) + G_c C_p x_p(t_k) \tag{5-118}$$

for k = 0,1,..., and for controller number 2,



$\tau + \delta < T$

$0 < \tau < T - \delta_c$

FIGURE 28 THE TIME RESPONSES OF y_{C1} and y_{C2}

$$x_{c2}(t_k+1+\tau+\delta_c) = F_c x_{c2}(t_k+\tau+\delta_c) + G_c C_p x_p(t_k+\tau) \tag{5-119}$$

$$y_{c2}(t_k+\tau+\delta_c) = H_c x_{c2}(t_k+\tau+\delta_c) + E_c C_p x_p(t_k+\tau) \tag{5-120}$$

for k = 0,1,...,

From Figure 28

$$u_p(t_k) = \frac{1}{2}\,[y_{c1}(t_k) + y_{c2}(t_k)] \tag{5-121}$$

where

$$y_{c1}(t_k) = y_{c1}(t_k+\delta_c) = H_c x_{c1}(t_k-1+\delta_c) + E_c C_p x_p(t_k-1) \tag{5-122}$$

and

$$y_{c2}(t_k) = y_{c2}(t_k-1+\delta_c+\tau) = H_c x_{c2}(t_k-1+\tau+\delta_c) + E_c C_p x_p(t_k-1+\tau) \tag{5-123}$$

Define new variables x_{hc1}, x_{hp1}, x_{hc2} and x_{hp2} as in type E, namely,

$$x_{hc1}(t_k+1+\delta_c) = x_{c1}(t_k+\delta_c) \tag{5-124}$$

$$x_{hp1}(t_k+1) = x_p(t_k) \tag{5-125}$$

$$x_{hc2}(t_k+1+\tau+\delta_c) = x_{c2}(t_k+\tau+\delta_c) \tag{5-126}$$

$$x_{hp2}(t_k+1+\tau) = x_p(t_k+\tau) \tag{5-127}$$

Then the equations of y_{c1} and y_{c2} become

$$y_{c1}(t_k-1+\delta_c) = H_c x_{hc1}(t_k+\delta_c) + E_c C_p x_{hp1}(t_k) \tag{5-128}$$

and

$$y_{c2}(t_k-1+\tau+\delta_c) = H_c x_{hc2}(t_k+\tau+\delta_c) + E_c C_p x_{hp2}(t_k+\tau) \tag{5-129}$$

Substituting (5-125) and (5-129) into (5-121) gives

$$u_p(t_k) = \frac{1}{2}\,[H_c x_{hc1}(t_k+\delta_c) + E_c C_p x_{hp1}(t_k) + H_c x_{hc2}(t_k+\tau+\delta_c) + E_c C_p x_{hp2}(t_k+\tau)] \tag{5-130}$$

at $t = t_k+\delta_c$, the value of u_p changes or let $t=t_o=t_k$. Then

$$x_p(t_k+\delta_c) = \Phi(\delta_c)x_p(t_k) + \psi_1(\delta_c)u_p(t_k) + \psi_2(\delta_c)w_p(t_k) \tag{5-131}$$

Substitution of (5-130) into (5-131) gives

$$\begin{aligned} x_p(t_k+\delta_c) = {} & \Phi(\delta_c)x_p(t_k) + \psi_1\frac{(\delta_c)}{2}\,E_c C_p x_{hp1}(t_k) + \frac{\psi_1(\delta_c)}{2}\,H_c x_{hc1}(t_k+\delta_c) \\ & + \frac{\psi_1(\delta_c)}{2}\,E_c C_p x_{hp2}(t_k+\delta_c) + \frac{\psi_1(\delta_c)}{2}\,H_c x_{hc2}(t_k+\tau+\delta_c) + \psi_2(\delta_c)w_p(t_k) \end{aligned} \tag{5-132}$$

at $t=t_k+\tau+\delta_c$ the value of u_p changes, or let $t=t_k+\tau+\delta_c$ and $t_o=t_k+\delta_c$. Then

$$x_p(t_k+\tau+\delta_c) = \Phi(\tau)x_p(t_k+\delta_c) + \psi_1(\tau)u_p(t_k+\delta_c) + \psi_2(\tau)w_p(t_k) \tag{5-133}$$

From Figure 28

$$u_p(t_k+\delta_c) = \frac{1}{2}\,[y_{c1}(t_k+\delta_c) + y_{c2}(t_k+\delta_c)] \tag{5-134}$$

where

$$y_{c1}(t_k+\delta_c) = H_c x_{c1}(t_k+\delta_c) + E_c C_p x_p(t_k) \tag{5-135}$$

and

$$y_{c2}(t_k+\delta_c) = y_{c2}(t_k-1+\tau+\delta_c) = H_c x_{hc1}(t_k+\tau+\delta_c) + E_c C_p x_{hp2}(t_k+\tau) \tag{5-136}$$

Substituting (5-135) and (5-136) into (5-134) gives

$$u_p(t_k+\delta_c) = \frac{1}{2}\,[H_c x_{c1}(t_k+\delta_c) + E_c C_p x_p(t_k) + H_c x_{hc2}(t_k+\tau+\delta_c) + E_c C_p x_{hp2}(t_k+\tau)] \tag{5-137}$$

Substituting (5-132) and (5-137) into (5-133) gives,

$$
\begin{aligned}
x_p(t_k+\tau+\delta_c) = {} & [\Phi(\tau+\delta_c) + \frac{\psi_1(\tau)}{2} E_cC_p]x_p(t_k) + \frac{\psi_1(\tau)}{2} H_cx_{c1}(t_k+\delta_c) \\
& + \Phi(\tau)\,\frac{\psi_1(\delta_c)}{2} E_cC_px_{hp1}(t_k) + \Phi(\tau)\,\frac{\psi_1(\delta_c)}{2} H_cx_{hc1}(t_k+\delta_c) \\
& + [\Phi(\tau)\,\frac{\psi_1(\delta_c)}{2} E_cC_p + \frac{\psi_1(\tau)}{2} E_cC_p]x_{hp2}(t_k+\tau) + [\Phi(\tau) + \psi_1\frac{(\delta_c)}{2} H_c \\
& + \psi_1\frac{(\delta_c)}{2} H_c]x_{hc2}(t_k+\tau+\delta_c) + [\Phi(\tau)\ {}_2(\delta_c) + \psi_2(\tau)]w_p(t_k)
\end{aligned}
\tag{5-138}
$$

The equation of $x_p(t_k+1)$ can be derived by letting $t=t_k+1$ and $t_o=t_k+\tau+\delta_c$. Then

$$
x_p(t_k+1) = \Phi(T-\tau-\delta_c)x_p(t_k+\tau+\delta_c) + \psi_1(T-\tau-\delta_c)u_p(t_k+\tau+\delta_c) + \psi_2(T-\tau-\delta_c)w_p(t_k) \tag{5-139}
$$

Consider $u_p(t_k+\tau+\delta_c)$.

From Figure 28

$$
u_p(t_k+\tau+\delta_c) = \frac{1}{2}\,[y_{c1}(t_k+\delta_c) + y_{c2}(t_k+\tau+\delta_c)] \tag{5-140}
$$

Substituting (5-118) and (5-120) into (5-140) gives

$$
\begin{aligned}
u_p(t_k+\tau+\delta_c) = \frac{1}{2}\,[& H_cx_{c1}(t_k+\delta_c) + E_cC_px_p(t_k) + H_cx_{c2}(t_k+\tau+\delta_c) \\
& + E_cC_px_p(t_k+\tau)]
\end{aligned}
\tag{5-141}
$$

Substituting (5-138) and (5-141) into (5-139) gives

$$x_p(t_k+1) = \{\Phi(T-\tau-\delta_c)[\Phi(\tau+\delta_c) + \psi_1\frac{(\tau)}{2} E_cC_p] + \psi_1\frac{(T-\tau-\delta_c)}{2} E_cC_p\} x_p(t_k)$$

$$+ [\Phi\frac{(T-\tau-\delta_c)}{2} \psi_1(\tau)H_c + \psi_1\frac{(T-\tau-\delta_c)}{2} H_c]x_{c1}(t_k+\delta_c) + \Phi(T-\delta_c)\psi_1\frac{(\delta_c)}{2} E_cC_px_{hp1}(t_k)$$

$$+ \Phi(T-\delta_c)\psi_1\frac{(\delta_c)}{2} H_cx_{hp1}(t_k+\delta_c) + \Phi(T-\tau-\delta_c)[\Phi(\tau)\frac{\psi_1(\delta_c)}{2}E_cC_p$$

$$+ \psi_1\frac{(\tau)}{2} E_cC_p]x_{hp2}(t_k+\tau) + \Phi(T-\tau-\delta_c)[\Phi(\tau)\frac{\psi_1(\delta_c)}{2} H_c + \frac{\psi_1(\tau)}{2} H_c]x_{hc2}(t_k+\tau+\delta_c)$$

$$+ \psi_1\frac{(T-\tau-\delta_c)}{2} H_cx_{c2}(t_k+\tau+\delta_c) + \psi_1\frac{(T-\tau-\delta_c)}{2} E_cC_px_p(t_k+\tau)$$

$$+ \{\Phi(T-\tau-\delta_c)[\Phi(\tau)\psi_2(\delta_c) + \psi_2(\tau)] + \psi_2(T-\tau-\delta_c)\}\ w_p(t_k) \qquad (5\text{-}142)$$

where

$$x_p(t_k+\tau) = \Phi(T)x_p(t_k) + \psi_1\frac{(\tau)}{2} E_cC_px_{hp1}(t_k) + \psi_1\frac{(\tau)}{2} H_cx_{hc1}(t_k+\delta_c)$$

$$+ \psi_1\frac{(\tau)}{2} E_cC_px_{hp2}(t_k+\tau) + \psi_1\frac{(\tau)}{2} H_cx_{hc2}(t_k+\tau+\delta_c)$$

$$+ \psi_2(\tau)w_p(t_k) \qquad (5\text{-}143)$$

Substituting (5-143) into (5-119) and (5-120) gives

$$x_{c2}(t_k+1+\tau+\delta_c) = F_cx_{c2}(t_k+\tau+\delta_c) + G_cC_p\ (\tau)x_p(t_k) + G_cC_p\psi_1\frac{(\tau)}{2}E_cC_px_{hp1}(t_k)$$

$$+ G_cC_p\psi_1\frac{(\tau)}{2} H_cx_{hc1}(t_k+\delta_c) + G_cC_p\psi_1\frac{(\tau)}{2} E_cC_px_{hp2}(t_k+\tau)$$

$$+ G_cC_p\psi_1\frac{(\tau)}{2} H_cx_{hc2}(t_k+\tau+\delta_c) + G_cC_p\psi_2(\tau)\ w_p(t_k) \qquad (5\text{-}144)$$

$$y_{c2}(t_k+\tau+\delta_c) = H_c x_{c2}(t_k+\tau+\delta_c) + E_c C_p\ (\tau) x_p(t_k) + E_c C_p \psi_1 \frac{(\tau)}{2} E_c C_p x_{hp1}(t_k)$$

$$+ E_c C_p \psi_1 \frac{(\tau)}{2} H_c x_{hc1}(t_k+\delta_c) + E_c C_p \psi_1 \frac{(\tau)}{2} E_c C_p x_{hp2}(t_k+\tau)$$

$$+ E_c C_p \psi_1 \frac{(\tau)}{2} H_c x_{hc2}(t_k+\tau+\delta_c) + E_c C_p \psi_2(\tau) w_p(t_k) \qquad (5\text{-}145)$$

These equations can be put in compact form by writing them in terms of a combined stated vector.

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{hp1}(t_k) \\ x_{c1}(t_k+\delta_c) \\ x_{hc1}(t_k+\delta_c) \\ x_{hp2}(t_k+\tau) \\ x_{c2}(t_k+\tau+\delta_c) \\ x_{hc2}(t_k+\tau+\delta_c) \end{bmatrix} \quad , x(t_k+1) = \begin{bmatrix} x_p(t_k+1) \\ x_{hp1}(t_k+1) \\ x_{c1}(t_k+1+\delta_c) \\ x_{hc1}(t_k+1+\delta_c) \\ x_{hp2}(t_k+1+\tau) \\ x_{c2}(t_k+1+\tau+\delta_c) \\ x_{hc2}(t_k+1+\tau+\delta_c) \end{bmatrix}$$

The state equations become

$$x(t_k+1) = F(T,\tau) x(t_k) + G(T,\tau) w_p(t_k) \qquad (5\text{-}146)$$

where $F(T,\tau)$ is

$$f_{11} = \Phi(T-\tau-\delta_c)[\Phi(\tau+\delta_c) + \psi_1 \frac{(\tau)}{2} E_c C_p] + \psi_1 \frac{(T-\tau-\delta_c)}{2} E_c C_p + \psi_1 \frac{(T-\tau-\delta_c)}{2} E_c C_p\ (\tau)$$

$$f_{12} = \Phi(T-\delta_c) \psi_1 \frac{(\delta_c)}{2} E_c C_p + \psi_1 \frac{(T-\tau-\delta_c)}{2} E_c C_p \psi_1 \frac{(\tau)}{2} E_c C_p$$

$$f_{13} = \Phi(T-\tau-\delta_c) \psi_1 \frac{(\tau)}{2} H_c$$

$$f_{14} = \Phi(T-\delta_c) \psi_1 \frac{(\delta_c)}{2} H_c + \psi_1 \frac{(T-\tau-\delta_c)}{2} E_c C_p \psi_1 \frac{(\tau)}{2} H_c$$

$$f_{15} = \Phi(T-\tau-\delta_c)[\Phi(\tau)\psi_1\frac{(\delta_c)}{2}\, E_cC_p + \psi_1\frac{(\tau)}{2}\, E_cC_p] + \psi_1\frac{(T-\tau-\delta_c)}{2}\, E_cC_p\psi_1\frac{(\tau)}{2}\, E_cC_p$$

$$f_{16} = \psi_1\frac{(T-\tau-\delta_c)}{2}\, H_c$$

$$f_{17} = \Phi(T-\tau-\delta_c)[\Phi(\tau)\psi_1\frac{(\delta_c)}{2}\, H_c + \psi_1\frac{(\tau)}{2}\, H_c] + \psi_1\frac{(T-\tau-\delta_c)}{2}\, E_cC_p\psi_1\frac{(\tau)}{2}\, H_c$$

$$f_{21} = 1$$

$$f_{22} = f_{25} = f_{24} = f_{25} = f_{26} = f_{27} = 0$$

$$f_{31} = G_cC_p$$

$$f_{32} = 0$$

$$f_{33} = F_c$$

$$f_{34} = f_{35} = f_{36} = f_{37} = 0$$

$$f_{41} = f_{42} = 0$$

$$f_{43} = 1$$

$$f_{44} = f_{45} = f_{46} = f_{47} = 0$$

$$f_{51} = \Phi(\tau)$$

$$f_{52} = \psi_1\frac{(\tau)}{2}\, E_cC_p$$

$$f_{53} = 0$$

$$f_{54} = \psi_1\frac{(\tau)}{2}\, H_c$$

$$f_{55} = \psi_1\frac{(\tau)}{2}\, E_cC_p$$

$$f_{56} = 0$$

$$f_{57} = \psi_1\frac{(\tau)}{2}\, H_c$$

$$f_{61} = G_cC_p\, f_{51}$$

$$f_{62} = G_cC_p\, f_{52}$$

$$f_{63} = 0$$

$$f_{64} = G_cC_p\, f_{54}$$

$$f_{65} = G_cC_p\, f_{55}$$

$$f_{66} = F_c$$

$$f_{67} = G_cC_p\, f_{57}$$

$$f_{71} = f_{72} = f_{73} = f_{74} = f_{75} = 0$$

$$f_{76} = 1$$

$$f_{77} = 0$$

and $G(T,\tau)$ is

$$g_1 = \Phi(T-\tau-\delta_c)[\Phi(\tau)\psi_2(\delta_c) + \psi_2(\tau)] + \psi_2(T-\tau-\delta_c) + \psi_1\frac{(T-\tau-\delta_c)}{2}\, E_cC_p{}_2(\tau)$$

$$g_2 = 0$$

$$g_3 = 0$$

$$g_4 = 0$$

$$g_5 = \psi_2(\tau)$$

$$g_6 = G_cC_p\psi_2(\tau)$$

$$g_7 = 0$$

The controller output equations are

$$y_{c1}(t_k+\delta_c) = H_1x(t_k) \tag{5-147}$$

and

$$y_{c2}(t_k+\tau+\delta_c) = H_2x(t_k) + o \tag{5-148}$$

then

$$H_1 = [E_cC_p \quad 0 \quad H_c \quad 0 \quad 0 \quad 0 \quad 0] \tag{5-149}$$

$$H_2 = [E_cC_p\,(\tau) \quad E_cC_p\,{}_{12}\left(\frac{\tau}{2}\right)E_cC_p \quad 0 \quad E_cC_p\,{}_{12}\left(\frac{\tau}{2}\right)E_cC_p \quad H_c \quad E_cC_p\,{}_{12}\left(\frac{\tau}{2}\right)H_c] \tag{5-150}$$

and

$$\rho = E_cC_p\,{}_2(\tau) \tag{5-151}$$

The same as variations D and E

$$e_A(t) = (H_1 - H_2)x(t_k) - \rho w_p(t_k) \tag{5-152}$$

for $t_k+\tau+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,\ldots,$ $0\leq\tau<T-\delta_c$, and

$$e_B(t) = (H_1F - H_2)x(t_k) + (H_1G-\rho)w_p(t_k) \tag{5-153}$$

for $t_k+1+\delta_c \leq t < t_k+1+\tau+\delta_c$, $k = 0,1,\ldots,$ $0<\tau\leq T-\delta_c$.

Figure 29 shows the skewed sampling and inherent errors.

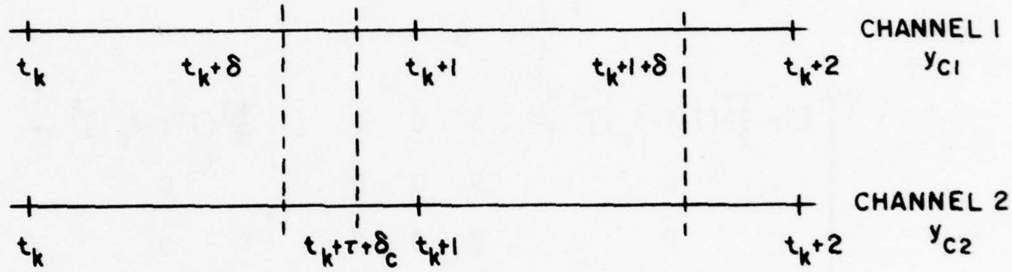
5.3.1 Covariance Analysis and Example

The analysis is the same as that for variation D.

5.3.2 Example

With data from the first example shown in section II, the matrix $F(T,\tau)$ and $G(T,\tau)$

$$F = \begin{bmatrix} (1-\frac{kT}{2} - k(T-\tau-\delta_c) & \frac{-k\delta_c}{2} + \frac{k^2T}{4}(T-\tau-\delta_c) & 0 & 0 & (\frac{-k\delta_c}{2} - \frac{kT}{2}) + \frac{k^2T}{2}(T-\tau-\delta_c) & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & \frac{-kT}{2} & 0 & 0 & \frac{-kT}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



$$\ell_A(t) = y_{C1}(t_k+\delta_c) - y_{C2}(t_k+\tau+\delta_c)$$

FOR $t_k+\tau+\delta_c \le t < t_k+1+\delta_c$ $\quad k = 0,1,\dots,\quad 0 \le \tau < T-\delta$

$$\ell\ (t) = y_{C1}(t_k+1+\delta_c) - y_{C2}(t_k+\tau+\delta_c)$$

FOR $t_k+1+\delta_c \le t < t < t_k+1+\tau+\delta_c$ $\quad k=0,1,\dots\quad 0 < \tau \le T-\delta$

FIGURE 29 SKEWED SAMPLING AND INHERENT ERRORS

$$G = \begin{bmatrix} T - \frac{kT}{2}(T-\tau-\delta_c) \\ 0 \\ 0 \\ 0 \\ \tau \\ 0 \\ 0 \end{bmatrix}$$

$$Gw_kG^T = \begin{bmatrix} [T-\frac{kT}{2}(T-\tau-\delta_c)]^2 \frac{\sigma_w^2}{T} & 0 & 0 & 0 & [T-\frac{kT}{2}(T-\tau-\delta_c)]^2 \frac{\sigma_w^2}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ [T-\frac{kT}{2}(T-\tau-\delta_c)] \frac{\sigma_w^2 T}{T} & 0 & 0 & 0 & \frac{\sigma_w^2 T^2}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By using (5-151), (5-152) and (5-153)

$$H_1 = [-k \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$H_2 = [-k \quad \frac{k^2T}{2} \quad 0 \quad 0 \quad \frac{k^2T}{2} \quad 0 \quad 0]$$

$$(H_1 - H_2 = [0 \quad \frac{-k^2T}{2} \quad 0 \quad 0 \quad \frac{-k^2T}{2} \quad 0 \quad 0]$$

$$H_1F = [-k\,(1 + \frac{kT}{2} - \frac{kT}{2} - \frac{k\delta_c}{2}) \quad -k[\frac{-k\delta_c}{2} + \frac{k^2}{4}(T-\tau-\delta_c)] \quad 0 \quad 0$$

$$-k[\frac{\delta_c}{2} - \frac{kT}{2} + \frac{k^2T}{4}(T-\tau-\delta_c)] \quad 0 \quad 0$$

$$(H_1F-H_2) = [\frac{-k^2T}{2} + \frac{k^2T}{2} + \frac{k^2\delta_c}{2} - k[\frac{-k\delta_c}{2} + \frac{k^2T}{4}(T-\tau-\delta_c)] \quad 0 \quad 0$$

$$- k[\frac{-k\delta_c}{2} + \frac{k^2T}{4}(T-\tau-\delta_c)] \quad 0 \quad 0]$$

$$\rho = -kT$$

$$H_1G = -k(T - \frac{kT}{2}(T-\tau-\delta_c)$$

$$H_1G-\rho = -kT + \frac{k^2T}{2}(T-\tau-\delta_c) + kT = -k(T-\tau) + \frac{k^2T}{2}(T-\tau-\delta_c)$$

$$P_{eAss} = \frac{\sigma_w^2 k^4 \tau^2}{4} [\frac{\tau}{T} \frac{[T- \frac{kT}{2}(T-\tau-\delta_c)]}{1-[1-kT+ \frac{k^2T}{2}(T-\tau-\delta_c)]^2} + \frac{\tau^2}{T}(1-(1-kT)^2]] + \frac{k^2T^2\sigma_w^2}{T}$$

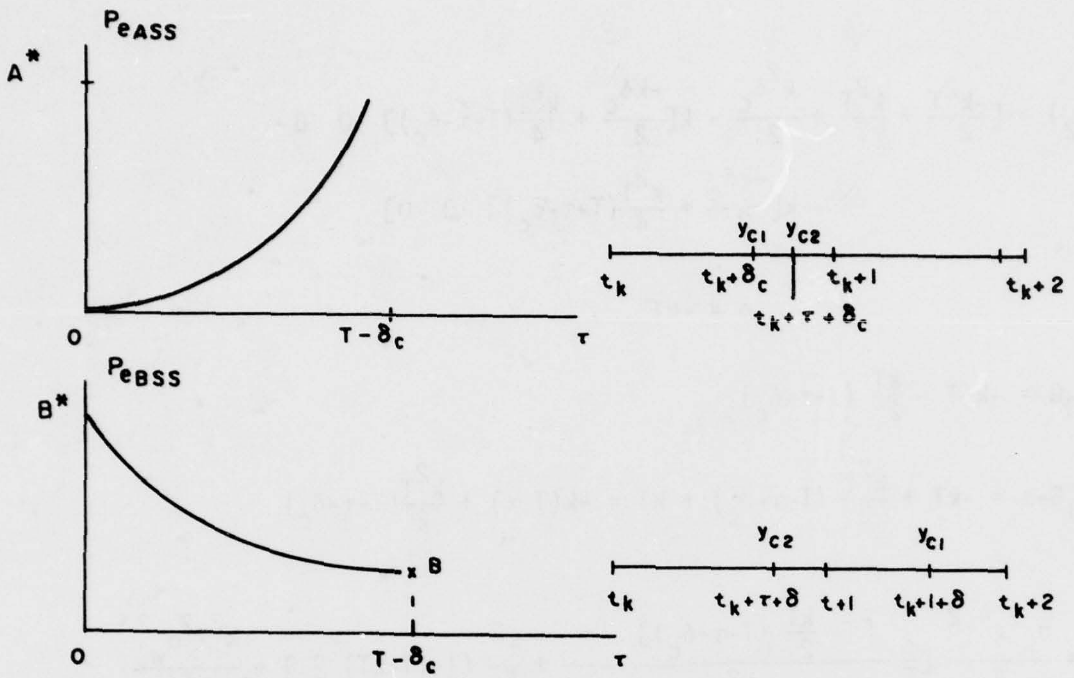
and

$$P_{eBss} = \frac{k^4}{4} \frac{[T+\delta_c-\tau]^2[T-kT\ 2(T-\tau-\delta_c)]^2}{1-[1-kT+\frac{k^2T}{2}(T-\tau-\delta_c)]^2} \frac{\sigma_w^2}{T} + 2[\frac{k^2\delta_c - \frac{k^2T}{2}(T-\tau+\delta_c][\frac{k^2T}{2} + \frac{k^2\delta_c}{2} - \frac{k^2T}{2}]}{1-[1-kT + \frac{k^2T}{2}(T-\tau-\delta_c)][1-kT]}$$

$$[T - \frac{kT}{2}(T-\tau-\delta_c)] \frac{\tau\sigma_w^{4\ 2}}{T}]$$

$$+ \frac{[- \frac{k^2\delta_c}{2} - \frac{k^3T}{4}(T-\tau-\delta_c)]^2 \frac{\sigma_w^2T^2}{T} + [-k(T-\tau) + \frac{k^2T}{2}(T-\tau-\delta_c)]^2 \frac{\sigma_w^2T}{T}}{1-[1-kT]^2}$$

P_{eAss} and P_{eBss} are plotted in Figure 30 as a function of τ. The diagrams to the right of each plot show the times corresponding to the values of the controller outputs used to calculate e_A and e_B. The results obtained for this example agree with intuition. For small τ, the variance of e_A is small and that for e_B is large. For large τ the complementary situation holds.



WHERE

$$A^* = \frac{\sigma_w^2 k^4 (T-\delta)^2}{4}\left[\frac{(T-\delta_c)}{kT(2-kT)} + \frac{(T-\delta_c)^2}{T\left\{1-\left[1-k(T-\delta_c)\right]^2\right\}}\right] + k\,\frac{(T_k-\delta_c)^2\sigma_w^{\,2}}{T}$$

$$B^* = \frac{k^3}{4}\;\frac{(T+\delta_c)^2\delta_w^2}{(2-kT)} + k^2 T\sigma_w^2$$

$$B = \frac{k^3\delta_c^2\sigma_w^2}{2-kT} + \frac{k^4\delta_c^2(T-\delta_c)^2\sigma_w}{T\left\{1-(1-kT)\left[1-k(T-\delta_c)\right]\right\}} + \frac{k^2\delta_c(T-\delta_c)^2\sigma_w^2}{2T\left\{1-\left[1-k(T-\delta)\right]^2\right\}}\;\underline{k^2\delta_c^2\sigma_w^2}$$

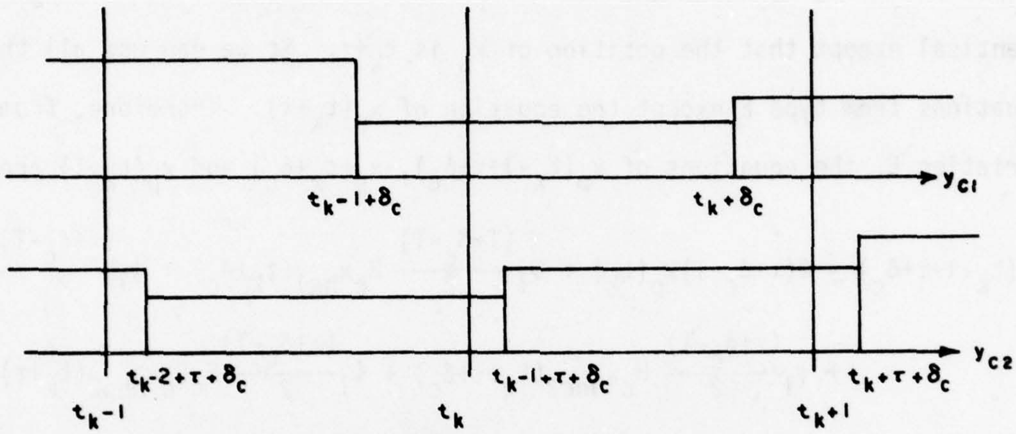
FIGURE 30 P_{eASS} and P_{eBSS} $0 < \tau < T-\delta_c$

5.3.3 Variation F (Case II): System Configuration and Dynamic Equations

Figure 31 shows the time responses of y_{c1} and y_{c2}. From this figure, $x_p(t_k+\tau)$ must be greater than $x_p(t_k-1+\tau+\delta_c)$ because $\delta_c<T$. For example if $\tau=0$ then $x_p(t_k-1+\tau+\delta_c)$ becomes $x_p(t_k-1+\delta_c)$ *which is less than* $x_p(t_k)$. If we compare this figure and Figure 25 of variation E. These time responses are identical except that the position of x_p is $t_k+\tau$. So we can use all the equations from type E except the equation of $x_p(t_k+\tau)$. Therefore, from *variation* E, the equations of $x_p(t_k-1+\tau+\delta_c)$, $x_p(t_k+\delta_c)$ and $x_p(t_k+1)$ are:

$$\begin{aligned} x_p(t_k-1+\tau+\delta_c) = {} & \Phi(\tau+\delta_c-T)x_p(t_k) + \psi_1\frac{(T+\delta_c-T)}{2} H_c x_{hc1}(t_k+\delta_c) + \psi_1\frac{(\tau+\delta_c-T)}{2}E_cC_p x_{hp1}(t_k) \\ & + \psi_1\frac{(\tau+\delta_c-T)}{2} H_c x_{hhc2}(t_k+\tau+\delta_c) + \psi_1\frac{(\tau+\delta_c-T)}{2}E_cC_p x_{hhp2}(t_k+\tau) \\ & + \psi_2(\tau+\delta_c-T)w_p(t_k) \end{aligned} \tag{5-154}$$

$$\begin{aligned} x_p(t_k+\delta_c) = {} & \Phi(\delta_c)x_p(t_k) + [\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} H_c + \psi_1\frac{(T-\tau)}{2}H_c]\, x_{hc1}(t_k+\delta_c) \\ & + [\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} E_cC_p + \psi_1\frac{(T-\tau)}{2}E_cC_p]x_{hp1}(t_k) \\ & + \Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}H_c x_{hhc2}(t_k+\tau+\delta_c) + \Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}E_cC_p x_{hhp2}(t_k+\tau) \\ & + \psi_1(T-\tau)H_c x_{hc2}(t_k+\tau+\delta_c) + \psi_1\frac{(T-\tau)}{2}E_cC_p x_{hp2}(t_k+\tau) \\ & + [\Phi(T-\tau)\psi_2(\tau+\delta_c-T) + \psi_2(T-\tau)]w_p(t_k) \end{aligned} \tag{5-155}$$



$\tau + \delta_c > T$

$T - \delta_c < \tau < \delta_c$

FIGURE 31 THE TIME RESPONSES OF y_{C1} and y_{C2}

$$x_p(t_k+1) = [\Phi(T) + \psi_1\frac{(T-\delta_c)}{2}E_cC_p]x_p(t_k) + \Phi(T-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}H_c$$

$$+ \psi_1\frac{(T-\tau)}{2}H_c]x_{hc1}(t_k+\delta_c) + \Phi(T-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}E_cC_p$$

$$+ \psi_1\frac{(T-\tau)}{2}E_cC_p]x_{hp1}(t_k) + \Phi(2T-\delta_c-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}H_cx_{hhc2}(t_k+\tau+\delta_c)$$

$$+ \Phi(2T-\delta_c-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2}E_cC_px_{hhp2}(t_k+\tau) + [\Phi\ (T-\delta_c)\psi_1\frac{(T-\tau)}{2}H_c$$

$$+ \psi_1\frac{(T-\delta_c)}{2}H_c]x_{hc2}(t_k+\tau+\delta_c) + [\Phi(T-\delta_c)\psi_1\frac{(T-\tau)}{2}E_cC_p$$

$$+ \psi_1\frac{(T-\delta_c)}{2}E_cC_p]x_{hp2}(t_k+\tau) + \psi_1\frac{(T-\delta_c)}{2}H_cx_{c1}(t_k+\delta_c)$$

$$+ \{\Phi(T-\delta_c)[\Phi(T-\tau)\psi_2(\tau+\delta_c-T) + \psi_2(T-\tau)] + \psi_2(T-\delta_c)\}w_p(t_k) \qquad (5\text{-}156)$$

The same as variation F (case I) the discrete-time equations for controller number one are:

$$x_{c1}(t_k+1+\delta_c) = F_cx_{c1}(t_k+\delta_c) + G_cC_px_p(t_k) \qquad (5\text{-}157)$$

$$y_{c1}(t_k+\delta_c) = H_cx_{c1}(t_k+\delta_c) + E_cC_px_p(t_k) \qquad (5\text{-}158)$$

for $k = 0,1,\ldots,$ and for controller number 2,

$$x_{c2}(t_k+1+\tau+\delta_c) = F_cx_{c2}(t_k+\tau+\delta_c) + G_cC_px_p(t_k+\tau) \qquad (5\text{-}159)$$

$$y_{c2}(t_k+\tau+\delta_c) = H_cx_{c2}(t_k+\tau+\delta_c) + E_cC_px_p(t_k+\tau) \qquad (5\text{-}160)$$

for $k = 0,1,\ldots$.

Let $t=t_k+\tau$ and $t_o=t_k-1+\tau+\delta_c$ then by using (5-112)

$$x_p(t_k+\tau) = \Phi(T-\delta_c)x_p(t_k-1+\tau+\delta_c) + \psi_1(T-\delta_c)x_p(t_k-1+\tau+\delta_c) + \psi_2(T-\delta_c)w_p(t_k) \quad (5\text{-}161)$$

Substituting (5-79) and (5-84) into (5-162) gives

$$\begin{aligned}
x_p(t_k+\tau) = {} & \Phi(\tau)x_p(t_k) + [\Phi(T-\delta_c)\psi_1\frac{(\tau+\delta_c-T)}{2}H_c + \psi_1\frac{(T-\delta_c)}{2}H_c]\, x_{hc1}(t_k+\delta_c) \\
& + [\Phi(T-\delta_c)\psi_1\frac{(\tau+\delta_c-T)}{2}E_cC_p + \psi_1\frac{(T-\delta_c)}{2}E_cC_p]\, x_{hp1}(t_k) \\
& + \Phi(T-\delta_c)\psi_1\frac{(\tau+\delta_c-T)}{2}H_cx_{hhc2}(t_k+\tau+\delta_c) + \Phi(T-\delta_c)\psi_1\frac{(\tau+\delta_c-T)}{2}E_cC_px_{hhp2}(t_k+\tau) \\
& + \psi_1\frac{(T-\delta_c)}{2}H_cx_{hc2}(t_k+\tau+\delta_c) + \psi_1\frac{(T-\delta_c)}{2}E_cC_px_{hp2}(t_k+\tau) \\
& + [\Phi(T-\delta_c)\psi_2(\tau+\delta_c-T) + \psi_2(T-\delta_c)]w_p(t_k)
\end{aligned} \quad (5\text{-}162)$$

The inherent error is defined into two parts as follows:

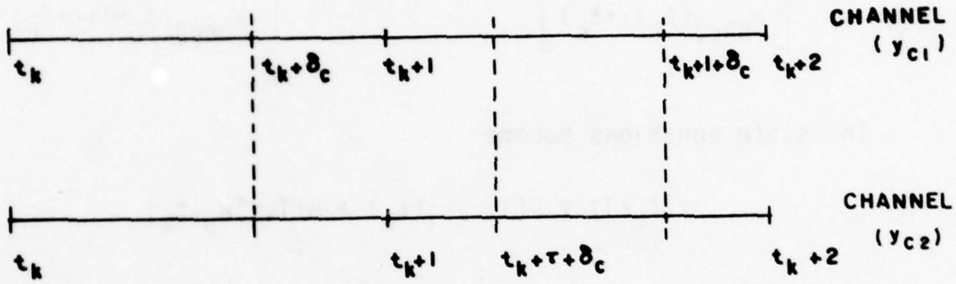
$$e_{A1}(t) = y_{c1}(t_k+\delta_c) - y_{c2}(t_k+\tau+\delta_c) \quad T-\delta_c \leq \tau < \delta_c \quad (5\text{-}163)$$

for $t_k+\tau+\delta_c \leq t < t_k+1+\delta_c$, $k = 0,1,\ldots$. And

$$e_{B1}(t) = y_{c1}(t_k+1+\delta_c) - y_{c2}(t_k+\tau+\delta_c) \quad T-\delta_c < \tau \leq \delta_c. \quad (5\text{-}164)$$

Figure 32 shows the skewed sampling and inherent errors.

These equations can be put in compact form by writing them in terms of a combined stated vector.



$$e_A(t) = y_{c1}(t_k+\delta_c) - y_{c2}(t_k+\tau+\delta_c)$$

FOR $t_k+\tau+\delta_c \le t < t_k+1+\delta_c$ $\quad k=0,1,\ldots,$ $\quad T-\delta_c \le \tau < \delta_c$

$$e_B(t) = y_{c1}(t_k+1+\delta_c) - y_{c2}(t_k+\tau+\delta_c)$$

FOR $t_k+1+\delta_c \le t < t_k+1+\tau+\delta_c$ $\quad k=0,1,\ldots,$ $\quad T-\delta_c < \tau \le \delta_c$

FIGURE 32 SKEWED SAMPLING AND INHERENT ERRORS

$$x(t_k) = \begin{bmatrix} x_p(t_k) \\ x_{hp1}(t_k) \\ x_{c1}(t_k+\delta_c) \\ x_{hc1}(t_k+\delta_c) \\ x_{hp2}(t_k+\tau) \\ x_{hhp2}(t_k+\tau) \\ x_{c2}(t_k+\tau+\delta_c) \\ x_{hc2}(t_k+\tau+\delta_c) \\ x_{hhc2}(t_k+\tau+\delta_c) \end{bmatrix}, x(t_k+1) = \begin{bmatrix} x_p(t_k+1) \\ x_{hp1}(t_k+1) \\ x_{c1}(t_k+1+\delta_c) \\ x_{hc1}(t_k+1+\delta_c) \\ x_{hp2}(t_k+1+\tau) \\ x_{hhp2}(t_k+1+\tau) \\ x_{c2}(t_k+1+\tau+\delta_c) \\ x_{hc2}(t_k+1+\tau+\delta_c) \\ x_{hhc2}(t_k+1+\tau+\delta_c) \end{bmatrix}$$

The state equations become

$$x(t_k+1) = F(T,\tau)x_1(t_k) + G(T,\tau)w_p(t_k) \tag{5-165}$$

where $F(T,\tau)$ is

$$f_{11} = \Phi(T) + \psi_1\frac{(T-\delta_c)}{2} E_cC_p$$

$$f_{12} = \Phi(T-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} E_cC_p + \psi_1\frac{(T-\tau)}{2} E_cC_p]$$

$$f_{13} = \psi_1\frac{(T-\delta_c)}{2} H_c$$

$$f_{14} = \Phi(T-\delta_c)[\Phi(T-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} H_c + \psi_1\frac{(T-\tau)}{2} H_c]$$

$$f_{15} = \Phi(T-\delta_c)\psi_1\frac{(T-\tau)}{2} E_cC_p + \psi_1\frac{(T-\delta_c)}{2} E_cC_p$$

$$f_{16} = \Phi(2T-\delta_c-T)\psi_1\frac{(\tau+\delta_c-T)}{2} E_cC_p$$

$$f_{17} = 0$$

$$f_{18} = \Phi(T-\delta_c)\psi_1\frac{(T-\tau)}{2} H_c + \psi_1\frac{(T-\delta_c)}{2} H_c$$

$$f_{19} = \Phi(2T-\delta_c-\tau)\psi_1\frac{(\tau+\delta_c-T)}{2} E_cC_p$$

$$f_{21} = 1$$

$$f_{22} = f_{23} = f_{24} = f_{25} = f_{26} = f_{27} = f_{28} = 0$$

$$f_{31} = G_c C_p$$

$$f_{32} = 0$$

$$f_{33} = F_c$$

$$f_{34} = f_{35} = f_{36} = f_{37} = f_{38} = f_{39} = 0$$

$$f_{41} = f_{42} = 0$$

$$f_{43} = 1$$

$$f_{44} = f_{45} = f_{46} = f_{47} = f_{48} = f_{49} = 0$$

$$f_{51} = \Phi(\tau)$$

$$f_{52} = \frac{\Phi(T-\delta_c)\psi_1(\tau+\delta_c-T)E_cC_p}{2} + \psi_1\frac{\Phi(T-\delta_c)E_cC_p}{2}$$

$$f_{53} = 0$$

$$f_{54} = \frac{\Phi(T-\delta_c)\psi_1(T+\delta_c-T)H_c}{2} + \psi_1\frac{\Phi(T-\delta_c)H_c}{2}$$

$$f_{55} = \psi_1\frac{(T-\delta_c)}{2}E_cC_p$$

$$f_{56} = \frac{\Phi(T-\delta_c)\psi_1(\tau+\delta_c-T)E_cC_p}{2}$$

$$f_{57} = 0$$

$$f_{58} = \psi_1(T-\delta_c)\ H_c$$

$$f_{59} = \frac{\Phi(T-\delta_c)\psi_1(\tau+\delta_c-T)\ H_c}{2}$$

$$f_{61} = f_{62} = f_{63} = f_{64} = 0$$

$$f_{65} = 1$$

$f_{66} = f_{67} = f_{68} = f_{69} = 0$

$f_{71} = G_c C_p\ f_{51}$

$f_{72} = G_c C_p\ f_{52}$

$f_{73} = 0$

$f_{74} = G_c C_p\ f_{54}$

$f_{75} = G_c C_p\ f_{55}$

$f_{76} = G_c C_p\ f_{56}$

$f_{77} = F_c$

$f_{78} = G_c C_p\ f_{58}$

$f_{79} = G_c C_p\ f_{59}$

$f_{81} = f_{82} = f_{83} = f_{84} = f_{85} = f_{86} = 0$

$f_{87} = 1$

$f_{88} = f_{89} = 0$

$f_{91} = f_{92} = f_{93} = f_{94} = f_{95} = f_{96} = f_{97} = 0$

$f_{98} = 1$

$f_{99} = 0$

$G_1(T,\tau)$ is

$g_1 = \Phi(T-\delta_c)[\Phi(T-\tau)\psi_2(\tau+\delta_c-T) + \psi_2(T-\tau)] + \psi_2(T-\delta_c]$

$g_2 = 0 = g_3 = g_4$

$g_5 = \psi_2(\tau)$

$g_6 = 0$

$g_7 = G_c C_p\ g_5$

$g_8 = g_9 = 0$

The controller output equations are

$$y_{c1}(t_k+\delta_c) = H_1 x(t_k) \tag{5-166}$$

and

$$y_{c2}(t_k+\tau+\delta_c) = H_2 x(t_k+\rho) \tag{5-167}$$

Then

$$H_1 = [E_c C_p \quad 0 \quad H_c \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \tag{5-168}$$

$$H_2 = [E_c C_p f_{51} \quad E_c C_p f_{52} \quad 0 \quad E_c C_p f_{54} \quad E_c C_p f_{55} \quad E_c C_p f_{56} \quad H_c \quad E_c C_p f_{58} \quad E_c C_p f_{59}] \tag{5-169}$$

and

$$\rho = E_c C_p \, g_5 \tag{5-170}$$

Using the same variations D and E,

$$e_A(t) = (H_1-H_2)x(t_k) - \rho w_k(t_k) \tag{5-171}$$

for $t_k+\tau+\delta_c \le t < t_k+1+\delta_c$, $k = 0,1,\ldots,$ $t-\delta_c \le \tau < \delta_c$, and

$$e_B(t_k) = (H_1F-H_2)x(t_k) + (H_1G-\rho)w_p(t_k) \tag{5-172}$$

for $t_k+1+\delta_c \le t < t_k+1+\tau+\delta_c$, $k = 0,1,\ldots,$ $t-\delta_c < \tau \le \delta_c$.

5.3.4 Covariance Analysis

The analysis is the same as in variation D.

5.3.5 Example

With the data from the first example of the basic model, the matrix F can be calculated as

$$
F = \begin{bmatrix}
\frac{1-k}{2}(T-\delta_c) & \frac{-k\delta_c}{2} & 0 & 0 & \frac{-k}{2}(2T-\tau-\delta_c) & \frac{-k}{2}(\tau+\delta_c-T) & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & \frac{-k\tau}{2} & 0 & 0 & \frac{-k}{2}(T-\delta_c) & \frac{-k}{2}(\tau+\delta_c-T) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix} \tag{5-173}
$$

$$
G = \begin{bmatrix} T \\ 0 \\ 0 \\ 0 \\ \tau \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5-174}
$$

$$P_{xss} = \begin{bmatrix} \frac{\sigma^2{}_w T}{kT(2-kT)} & 0 & 0 & 0 & \frac{\sigma^2{}_w T}{1-(1-kT)(1-k\tau)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sigma^2{}_w T}{1-(1-kT)(1-k\tau)} & 0 & 0 & 0 & \frac{\sigma^2{}_w T^2}{kT\tau(2-kT)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

From (5-171), (5-172) and (5-173) we have

$$H_1 = [-k \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$H_2 = [-k \quad \frac{k^2T}{2} \quad 0 \quad 0 \quad \frac{k^2}{2}(T-\delta_c) \quad \frac{k^2}{2}(\tau+\delta_c-T) \quad 0 \quad 0 \quad 0]$$

$$\rho = -kT$$

and

$$(H_1-H_2) = [0 \quad \frac{-k^2\tau}{2} \quad 0 \quad 0 \quad \frac{-k^2}{2}(T-\delta_c) \quad \frac{-k^2}{2}(\tau+\delta_c-T) \quad 0 \quad 0 \quad 0]$$

$$H_1F = [-k[1-\frac{k}{2}(T-\delta_c)] \quad \frac{k^2\delta_c}{2} \quad 0 \quad 0 \quad \frac{k^2}{2}(2T-\tau-\delta_c) \quad \frac{k^2}{2}(\tau+\delta_c-T) \quad 0 \quad 0 \quad 0]$$

$$H_1F-H_2 = [\frac{k^2}{2}(T-\delta_c) \quad \frac{k^2}{2}(\delta_c-\tau) \quad 0 \quad 0 \quad \frac{k^2}{2}(T-\tau) \quad 0 \quad 0 \quad 0 \quad 0]$$

$$H_1G = [-kT]$$

$$H_1G\text{-}\rho = -k(T-\tau)$$

$$P_{eAss} = \frac{k^4}{4}(T-\delta_c)^2 \frac{\sigma_w^2\tau}{kT(2-k\tau)} + \frac{k^2\tau^2\sigma_w^2}{T}$$

and

$$P_{eBss} = \frac{k^4}{4}\frac{(T-\delta_c)^2\sigma_w^2 T}{kT(2-kT)} + \frac{k^4}{4}\frac{(T-\tau)(T-\delta_c)\sigma_w^2\tau}{1-(1-kT)(1-k\tau)} + \frac{k^4}{4}\frac{(T-\tau)^2\sigma_w^2\tau}{kT(2-kT)} + \frac{k^2(T-\tau)^2\sigma_w^2}{T}$$

P_{eAss} and P_{eBss} are plotted in Figure 33 is a function of τ. The diagrams to the right of each plot show the times corresponding to the values of the controller outputs used to calculate e_A and e_B. The results agree with intuition just as in Case I. For small τ, the variance of e_A is small and that for e_B is large. For large τ, the complimentary situation holds.

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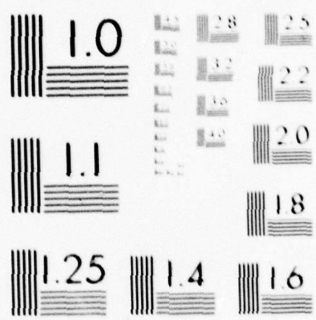
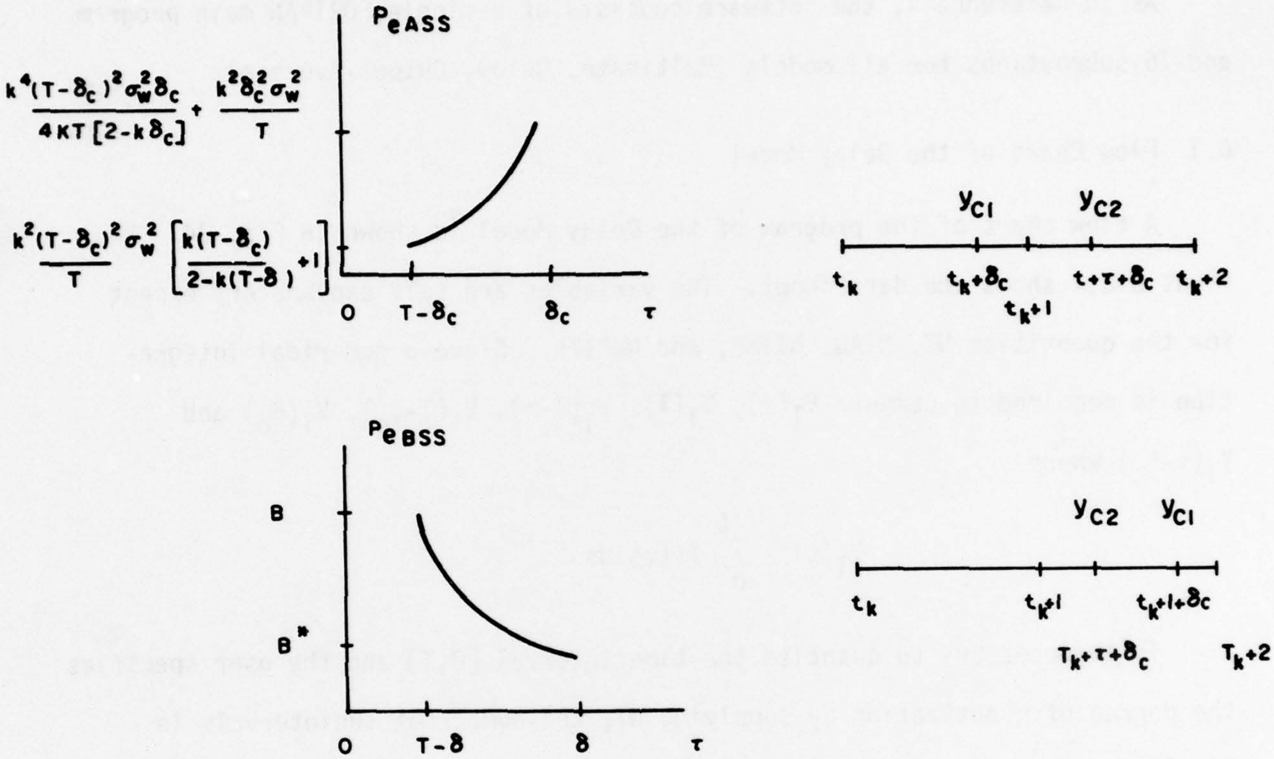


MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS 1963 A



$$B = k^3\sigma_w^2(T-\delta_c)\left[\frac{T-\delta}{4(2-kT)} + \frac{k\delta\,(T-\delta)}{2\left\{1-(1-kT)\left[1-k(T-\delta_c)\right]\right\}} + \frac{\delta_c}{T\left[2k(T-\delta_c)\right]}\right] + \frac{k^2\delta_c^2\sigma_w^2}{T}$$

$$B^* = k^2(T-\delta_c)^2\sigma_w^2\left[\frac{k}{4(2-kT)} - \frac{k^2\delta_c}{2\left\{1-(1-kT)(1-k\delta_c)\right\}} + \frac{k\delta_c}{4T(2-k\delta_c)} + \frac{1}{T}\right]$$

FIGURE 33 P_{eASS} and P_{eBSS} $T-\delta_c < \tau < \delta_c$

6.0 SOFTWARE FOR MODELING WITH COVARIANCE AND TRANSIENT-RESPONSE ANALYSIS

As in Reference 1, the software consists of a single FORTRAN main program and 16 subroutines for all models (Multirate, Delay, Output-Average).

6.1 Flow Chart of the Delay Model

A flow chart of the program of the Delay Model is shown in Fig. 34. The first block shows the data input. The variables are self explanatory except for the quantities NT, NTAU, NTIME, and NWRITE. Since a numerical integration is required to compute $V_1(\tau)$, $V_1(T)$, $V_1(T-\tau)$, $V_1(T-\delta_c)$, $V_1(\delta_c)$ and $V_1(\tau-\delta_c)$ where

$$V_1(t) = \int_0^t \Phi(t,s)ds$$

It is necessary to quantize the time interval [0,T] and the user specifies the degree of quantization by supplying NT, the number of subintervals in [0,T] which are to be used in the computation. The user specifies the number of evenly-spaced values of τ and δ_c by providing the number NTAU, and NDELAY; the values of τ and δ_c are then computed within the program as

$$\tau = \frac{NTAU*T}{NT}$$

and

$$DELAY = \frac{NDELAY*T}{NT}$$

NTIME determines the number of T-incremented points to be computed; that is, compute transient data for the following values of time.

$$TIME = 0,T,\dots, NTIME*T$$

NWRITE determines the values of TIME for which the data is to be written, that is, the time response data is to be written every NWRITE*T seconds

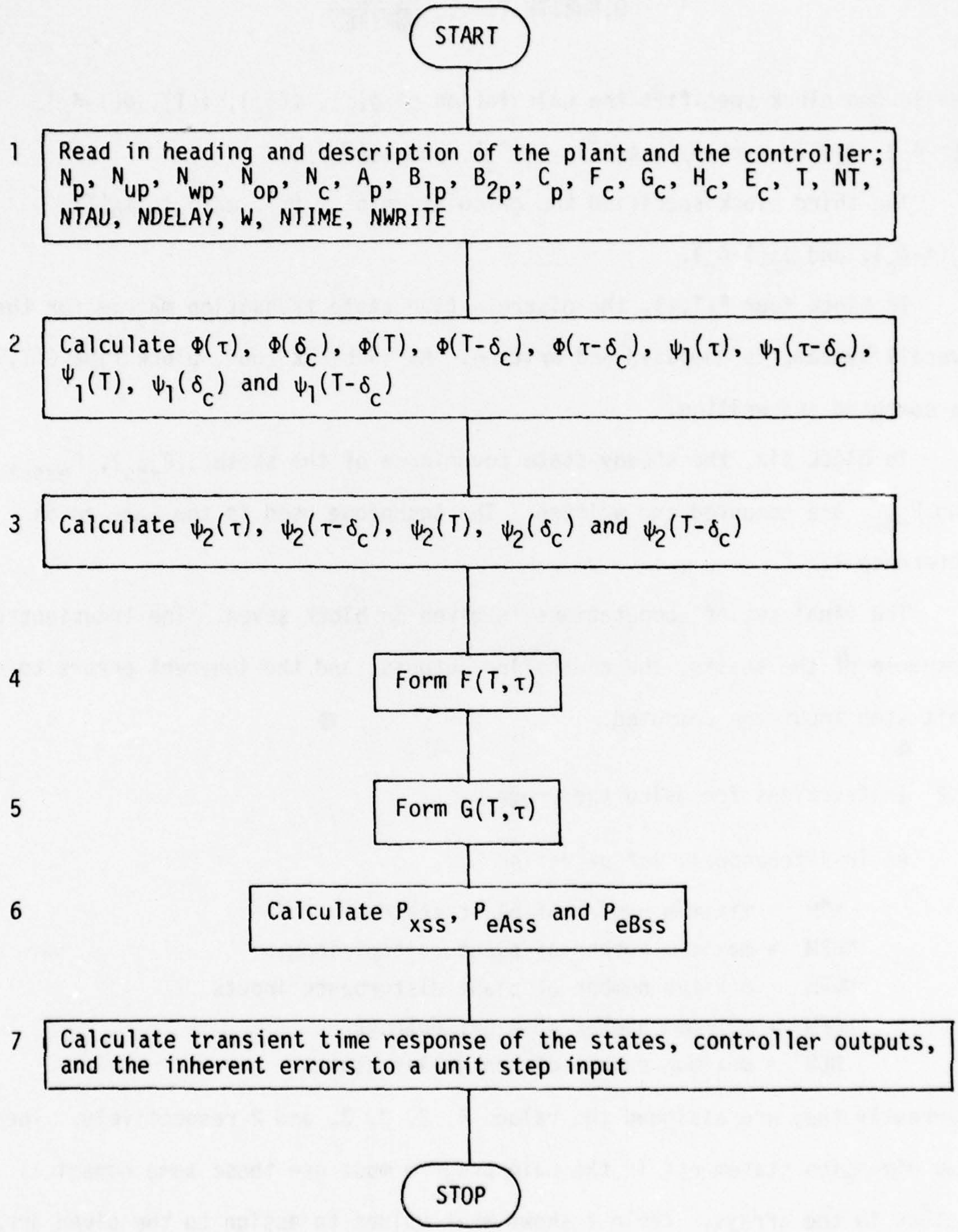


Figure 34. Flow Chart Describing the Major Computations of Program Skew

$$0, \text{NWRITE } T, \ldots, \frac{\text{NTIME}*T}{\text{NWRITE}}$$

The second block specifies the calculation of $\Phi(\tau)$, $\Phi(\delta_c)$, $\Phi(T)$, $\Phi(T-\delta_c)$, $\Phi(\tau-\delta_c)$, $\psi_1(\tau)$, $\psi_1(\tau-\delta_c)$, $\psi_1(T)$, $\psi_1(\delta_c)$, and $\psi_1(T-\delta_c)$.

The third block specified the calculation of $\psi_2(\tau)$, $\psi_2(\delta_c)$, $\psi_2(T)$, $\psi_2(\tau-\delta_c)$, and $\psi_2(T-\delta_c)$.

In block four $F(T,\tau)$, the discrete-time state transition matrix for the overall system, is computed and written. As in block four, block five $G(T,\tau)$ is computed and written.

In block six, the steady-state covariance of the states (P_{xss}), P_{eass}, and P_{eBss} are computed and written. The technique used is the same as in Reference 1.

The final set of computations is given in block seven. The transient time response of the states, the controller outputs, and the inherent errors to a unit step input are computed.

6.2 Instructions for Using the Program

As in Reference 1, let us define

NPM = maximum number of plant states
NUPM = maximum number of plant control inputs
NWPM = maximum number of plant disturbance inputs
NOPM = maximum number of plant outputs
NCM = maximum number of controller states.

Currently they are assigned the values 4, 2, 2, 3, and 2 respectively. The two dimension statements in the main program must use these same numerical values in the arrays. Table 2 shows what values to assign to the given arrays. In this table, the numerical values of NHM, NFM, and NRRM are calculated from

TABLE 2

Required Dimensions of All Arrays

AP(NPM,NPM)
B1P(NPM,NUPM)
B2P(NPM,NWPM)
CP(NOPM,NPM)
FC(NCM,NCM)
GC(NCM,NOPM)
HC(NUPM,NCM)
EC(NUPM,NOPM)
ECCP(NUPM,NPM)
GCCP(NCM,NPM)
PHITAU(NHM,NHM)
PHIT(NHM,NHM)
PHTATD(NHM,NHM)
PHTDEL(NHM,NHM)
PS1T(NHM,NHM)
PS1TAU(NHM,NHM)
PS1TAT(NHM,NHM)
PS1TTD(NHM,NHM)
PS1TDE(NHM,NHM)
PS2TAT(NHM,NHM)
PS2TTD(NHM,NHM)
PS2TDE(NHM,NHM)
PS2T(NHM,NHM)
PS2TAU(NHM,NHM)
F(NHM,NFM)
AD(NHM,NHM)
DD(NHM)
DELPHI(NHM,NHM)
HA(NUPM,NFM)
HB(NUPM,NFM)
E1(NUPM)
D2(NUPM)
X(NFM)
XW(NFM)

EIF(NHM,4)
EIV(NHM,4)
EH(NHM,NHM)
FHST(NHM,NHM)
KWA(NHM)
RF(NRRM)
RR(NRRM)
ID(20)
PT(NHM,NHM)
INDEX(NHM)
W(NWPM,NWPM)
PS(NHM,NHM)
V(NFM,NFM)
PXSS(NFM,NFM)
PEAXSS(NUPM,NUPM)
PEBXSS(NUPM,NUPM)
V1(NPM,NPM)
V1TAU(NPM,NPM)
V1TDEL(NPM,NPM)
V1TATD(NPM,NPM)
V1TTD(NPM,NPM)
H1(NUPM,NFM)
H2(NUPM,NFM)
AM(NFM,NFM)
PM(NFM,NFM)
D(NFM,NFM)
DW(NFM,NFM)
PL(NUPM,NUPM)
YC1(NUPM)
YC2(NUPM)
HPH(NUPM,NUPM)
GWG(NFM,NWPM)
PLB(NUPM,NUPM)
G(NFM,NWPM)

NHM = 2+NPM+NUPM

NFM = 2+NPM+3*NCM

NRRM = 4+NPM+4

There are two listings and three examples of output (A-7D pitch axis flight control systems) for the variations B and C in Appendix A. The flow chart in Figure 34 can be used for both of the variations except that for the equations of $F(T,\tau)$, $G(T,\tau)$, P_{xss}, P_{eAss}, and P_{eBss}. For variation B use the equations $F(T,\tau)$, $G(T,\tau)$, P_{xss}, P_{eAss}, and P_{eBss} in section 4.2 and for variation C use the equations of the matrices found in section 4.3.

7.0 SUMMARY

As mentioned in Section 1, there are three models which are developed from the Basic Model.

1. Multirate Model
2. Delay Model
3. Output-Average Model.

The Multirate Model allows for separate sampling rates for the external input and the digital controllers. If n in this model is equal to 1, then the Multirate Model is the same as the Basic Model except that the external input is sampled. The data from the first example of the Basic Model is applied to study the characteristics of the covariance errors of this model and the limit of the skew time is $0<\tau<\frac{T}{n}$. As discussed in the section of the Multirate Model, the covariance errors are the approximate value and they are less than the true value. The characteristics of the covariance errors follow the fact that, they are largest when the times at which y_{c1} and y_{c2} are farthest apart.

The Delay Model allows for computational delays due to the time required for data conversions and controls output computations. There are three variations A, B, and C. In variation A, there are three time delays (δ_a, δ_c and δ_d) which the sum of them is less than the skew time and the computations of y_{c1} and y_{c2} are completed in the same period (t_k, t_k+1). The limit of τ is $\Delta<\tau<T-\Delta$ where Δ is the sum of the delay. In variation B, there is one time delay (δ_c) which is less than the skew time and the computations of y_{c1} and y_{c2} aren't completed in the same period. The limit of τ is $\delta_c<\tau<T$. In variation C, there is also one time delay (δ_c) which is greater than the skew time τ and there are two cases for the computations of y_{c1} and y_{c2}. In Case I, the computations of y_{c1} and y_{c2} are completed in the same period and in Case II, the computations of y_{c1} and y_{c2} aren't completed in the same

period and the computation of y_{c2} are completed in the following period. There are no difference for the derivation of these two cases because the input of the plant is the first output of the controller. The difference is the value of the sum of τ and δ_c. The limit of the skew time are $0<\tau<\delta_c$. The data from the first example of the Basic Model is applied to these variations and the covariance errors follow the fact that, they are largest when the times at which y_{c1} and y_{c2} are farthest apart. The values of the covariance errors are approximate value and less than the true value which are the same as the Multirate Model. Only one of the expressions of the covariance errors depends on the time delay. This is because the data of the first example is the special ($A_p=0$). In the general case, both of the expressions of the covariance errors depend on the time delay.

The Output-Averaging Model provides for averaging the control outputs produced by each of the redundant controllers. There are three variations, D, E and F. In variation D, there is no time delay and the limit of the skew time is $0<\tau<T$. In variation E, there is one time delay δ_c which is less than τ. The same as variation B, the computations of y_{c1} and y_{c2} are not completed in the same period and the limit of the skew time is $\delta_c<\tau<T$. In variation F, which is the same as variation C, the time delay δ_c is greater than τ and there are two cases, Case I, the computations of y_{c1} and y_{c2} are completed in the same period and in Case II they aren't. Because the input of the plant is the average of the outputs of the controllers so the deviation of these two cases are different. The limit of τ of the first case is $0<\tau<T-\delta_c$ and the limit of τ of the second case is $T-\delta_c<\tau<\delta_c$. The data from the first example of the Basic Model is applied to these variations. The covariance errors follow the fact that, they are largest when the times at which y_{c1} and y_{c2} are farthest apart.

Section 6 describes the software which has been developed to apply the methods to realistic examples. A general description; a flow chart and user instructions are given for the Fortran program. The computer program listing and the example of the A-7D pitch-axis of the Multirate Model are given in Appendix B. Appendix A discusses and calculates the covariance of the sample-hold of the white gaussian noise.

Appendix A. The Correlation Function of the Sample and Zero-Order Hold of White Gaussian Noise

The white noise is defined as a stationary, zero-mean gaussian process with power spectrum

$$\mathcal{S}_w(f) \triangleq \frac{N_o}{2} \quad -\infty < f < \infty$$

where the dimensions of N_o are watts per cycle per second, or joules.

Actually, white noise (whether Gaussian or not) must be fictitious because its total mean power would be

$$N_w^2(t) = \int_{-\infty}^{\infty} \mathcal{S}_w(f)\, df = \infty \qquad \text{(C-2)}$$

which is not meaningful. Therefore, we can't sample zero-order hold white noise. There is the process which is equivalent with sample zero-order hold process from the fact that if we pass the white noise through a linear filter for which

$$\int_{-\infty}^{\infty} |H(f)|^2\, df < \infty \qquad \text{(C-3)}$$

produces at the filter output a stationary, zero-mean noise N(t). Then we have

$$\mathcal{S}_n(f) = \frac{N_o}{2} |H(f)|^2 \qquad \text{(C-4)}$$

and

$$\overline{N^2(t)} = \frac{N_o}{2} \int_{-\infty}^{\infty} |\, H(f)|^2 df. \qquad \text{(C-5)}$$

which from equation C-3, is finite.

Figure 1 is the block diagram of the equivalent process for sample-zero order hold process. The filter output is the average value of the noise in the interval T (sample period).

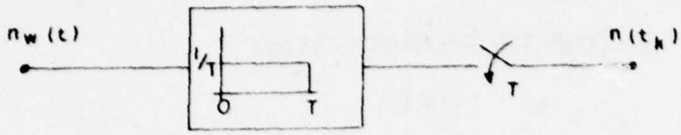


Figure 1

Let z(t) be the filter output, then

$$Z(t) = N_w(t) * h(t)$$

$$= \int_0^T N(t-\lambda)h(\lambda)d\lambda$$

$$= \frac{1}{T}\int_0^T N(t-\lambda)d\lambda$$

The correlation function of the filter output is

$$E[Z^2(t)] = E[\frac{1}{T^2}\int_0^T\int_0^T N_w(t-\lambda)h_w(t-\mu)d\lambda d\mu]$$

$$= \frac{1}{T^2}\int_0^T\int_0^T R_w(\lambda-\mu)d\lambda d\mu$$

where $R_n(t) \triangleq E[N_w(t)N_w(t+\tau)]$ and from equation C-1,

$$R_w(\tau) = \frac{N_o}{2}\delta(\tau)$$

Then

$$R_z(0) = \frac{1}{T^2}\int_0^T\int_0^T \frac{N_o}{2}\delta(\lambda-\mu)d\lambda d\mu = \frac{1}{T^2}\int_0^T \frac{N_o}{2}d\lambda d\mu = \frac{N_o}{2T}$$

Therefore the correlation function of the sample zero-order hold white Gaussian noise at $\tau=0$ is $\frac{N_o}{2T}$.

APPENDIX B

COMPUTER PROGRAM LISTING

FOR

PROGRAM SKEW

WRITTEN IN

FORTRAN

THE VARIATION A

LISTINGS

AND

EXAMPLES

```
      COMMON ELEMX,MAXI,MAXJ
      DIMENSION AP(4,4),B1P(4,2),B2P(4,2),CP(3,4),FC(2,2),
     1 HC(2,2),EC(2,3),PHIT(10,10),PSIT(10,10),F(14,14),
     2 DELPHI(10,10),EIP(10,4),EIV(10,4),FH(10,10),
     3 KWA(10),RF(20),RR(20),ID(20),ECCP(2,4),GCCP(2,4),
     4 INDEX(10),W(2,2),PS(10,10),PSITAT(10,10),GC(2,3),
     5 PSITTD(10,10),PHTATD(10,10),PHTDEL(10,10),AH(10,10),
     6 PHTTD(10,10),V1(4,4),V1TDEL(4,4),V1TATD(4,4),
     7 PS2TDE(10,10),PM(14,14),PS2TTD(10,10),V1TTD(4,4),
     8 AM(14,14),G(14,2),D(14,14),DW(14,14),PL(2,2),
     9 H1(2,14),H2(2,14),HA(2,14),HB(2,14),PLB(2,2),
     A PLWPL(2,2),PEAXSS(2,2),PEBXSS(2,2),PXSS(14,14),
     B FHST(10,10),DD(10),PT(10,10),PSITDE(10,10),
     C PHITAU(10,10),PS2TAT(10,10),GWG(14,14),
     D X(14),XW(14),E1(2),E2(2),EA(2),E11(2),E22(2),
     E YC1(2),YC2(2),PSITAU(10,10),V1TAU(4,4),HPH(2,2)
CCCCC PROVIDE MAXIMA FOR CALLED ARRAYS
      NPM = 4
      NUPM = 2
      NWPM = 2
      NOPM = 3
      NCM = 2
      NHM = 2*NPM + NUPM
      NFM = 2*NPM + 3*NCM
      NRRM = 4*NPM + 4
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCC READ INPUT DATA CCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,2222)
2222  FORMAT('1')
      READ(5,900) ID
  900 FORMAT(20A4)
      WRITE(6,902) ID
  902 FORMAT('1',20A4)
      READ(5,906)NP,NUP,NWP,NOP,NC
  906 FORMAT(5I3)
      WRITE(6,908) NP,NUP,NWP,NOP,NC
908   FORMAT('0NO. OF PLANT STATES =',I3/
     1 ' NO. OF PLANT INPUTS = ',I3/
     2 ' NO. OF DISTURBANCE INPUTS = ', I3/
     4 ' NO. OF PLANT OUTPUTS = ', I3/
     5 ' NO. OF CONTROLLER STATES (EACH CONTROLLER) = ',I3)
       WRITE(6,910)
  910 FORMAT('0PLANT STATE MATRIX -- AP')
  110 DO 112 I = 1,NP
      READ(5,914) (AP(I,J),J=1,NP)
112   WRITE(6,913) (AP(I,J),J=1,NP)
913   FORMAT(' ',8G13.6)
914   FORMAT(4F13.7)
915   FORMAT(7G13.6)
      WRITE(6,916)
  916 FORMAT('0PLANT CONTROL INPUT MATRIX -- B1P')
```

```
  120 DO 122 I = 1,NP
      READ(5,914)(B1P(I,J),J=1,NUP)
122   WRITE(6,913)(B1P(I,J),J=1,NUP)
      WRITE(6,918)
  918 FORMAT('0PLANT DISTURBANCE INPUT MATRIX -- B2P')
  130 DO 132 I=1,NP
      READ(5,914)(B2P(I,J),J=1,NWP)
132   WRITE(6,913)(B2P(I,J),J=1,NWP)
      WRITE(6,920)
  920 FORMAT('0PLANT OUTPUT MATRIX -- CP')
  140 DO 142 I=1,NOP
      READ(5,914)(CP(I,J),J=1,NP)
142   WRITE(6,913)(CP(I,J),J=1,NP)
      WRITE(6,922)
  922 FORMAT('0CONTROLLER STATE MATRIX -- FC')
  150 DO 152 I =1,NC
      READ(5,914)(FC(I,J),J=1,NC)
152   WRITE(6,913)(FC(I,J),J=1,NC)
      WRITE(6,924)
  924 FORMAT('0CONTROLLER CONTROL INPUT MATRIX -- GC ')
  160 DO 162 I=1,NC
      READ(5,914)(GC(I,J),J=1,NOP)
  162 WRITE(6,913)(GC(I,J),J=1,NOP)
      WRITE(6,925)
  925 FORMAT('0CONTROLLER OUTPUT MATRIX (STATES) -- HC')
  170 DO 172 I=1,NUP
      READ(5,914)(HC(I,J),J=1,NC)
  172 WRITE(6,913)(HC(I,J),J=1,NC)
      WRITE(6,926)
926   FORMAT('0CONTROLLER OUTPUT MATRIX (INPUTS) -- EC')
  180 DO 182 I=1,NUP
      READ(5,914)(EC(I,J),J=1,NOP)
  182 WRITE(6,913)(EC(I,J),J=1,NOP)
      READ(5,928)T,NT,NDELAY,NTAU
928   FORMAT(F10.4,5I5)
      XNT=NT
      XNTAU=NTAU
      XNDELA=NDELAY
      DELTA=T/XNT
      NIFTAU=NTAU-NDELAY
      NIFT=NT-NDELAY
      WRITE(6,930)T,NT,NDELAY,NTAU
930   FORMAT('1T = ',F10.4/
     1 ' NT = ',I5/
     2 ' NDELAY= ',I3//
     3 ' NTAU = ',I3//
     4 ' T = SAMPLE RATE.'/
     5 ' NT = NB. OF EVENLY-SPACED SUBINTERVALS INTO'/
     6 '        WHICH T IS DIVIDED.'//
     7 ' DELTA = T/NI/NT = INCREMENT USED IN THE'/
     8 '              NUMERICAL INTEGRATIONS.'//
      WRITE(6,931)
  931 FORMAT('0DISTURBANCE COVARIANCE MATRIX -- W')
  184 DO 186 I=1,NWP
```

```
      READ(5,914)(W(I,J),J=1,NWP)
  186 WRITE(6,913)(W(I,J),J=1,NWP)
      DO 850 I = 1,NWP
      DO 850 J = 1,NWP
850   W(I,J) = W(I,J)/T
      READ(5,888)IXTIME,NXTIME,NXWRIT
888   FORMAT(3I5)
      WRITE(6,988) IXTIME,NXTIME,NXWRIT
988   FORMAT(3I5)
      NII=NI-1
      NWRITE=NXWRIT
      NTIME=NXTIME
      ITIME=IXTIME
CCCCC CALCULATE ECCP AND GCCP  CCCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 701 I = 1,NUP
      DO 701 J = 1,NP
      ECCP(I,J) = 0.0
      DO 701 K = 1,NOP
701   ECCP(I,J) = ECCP(I,J) + EC(I,K)*CP(K,J)
      DO 702 I = 1,NC
      DO 702 J = 1,NP
      GCCP(I,J) = 0.0
      DO 702 K = 1,NOP
702   GCCP(I,J) = GCCP(I,J) + GC(I,K)*CP(K,J)
      DELHLF=DELTA/2.0
      TI=0
      DO 1 I=1,NP
      DO 1 J=1,NP
      V1(I,J)=0.0
      PT(I,J)=0.0
1     PS(I,J)=0.0
      IDEL=0
      DO 2 I=1,NP
2     V1(I,I) = DELHLF
      IMPULS = 0
      IPRINT=9
      DAPPRX=0.0
      EPS = 1.0E-7
      NTERMS=6
      IPOLE=0
      WMAX=-1.0
      DO 300 I = 1,NT
      CALL MADD(V1,PT,V1,NP,NPM,NHM,NPM)
      TI=TI+DELTA
      CALL EXPK2(AP,AH,B1P,DAPPRX,DD,DELPHI,EIF,EIV,EPS,
     1 PHIT,FH,FHST,PS1T,IMPULS,IPOLE,IPRINT,KWA,NHM,
     2 NRRM,NTERMS,NUP,NUPM,NP,NPM,RF,RR,TI,WMAX,INDEX)
      DO 3 II=1,NP
      DO 3 JJ=1,NP
3     PT(II,JJ)=PHIT(II,JJ)
      CALL MBYCON(DELHLF,PT,NP,NHM)
      CALL MADD(V1,PT,V1,NP,NPM,NHM,NPM)
      IDEL=IDEL+1
      IF(IDEL.EQ.NIFTAU)GO TO 4
```

```
      IF(IDEL.EQ.NDELAY) GO TO 7
      IF(IDEL.EQ.NTAU) GO TO 10
      IF(IDEL.EQ.NIFT)GO TO 12
      GO TO 300
4     DO 5 II=1,NP
      DO 5 JJ=1,NP
      PHTATD(II,JJ)=PHIT(II,JJ)
5     V1TATD(II,JJ) = V1(II,JJ)
      DO 6 II=1,NP
      DO 6 JJ=1,NUP
6     PSITAT(II,JJ)=PSIT(II,JJ)
      GO TO 300
7     DO 8 II=1,NP
      DO 8 JJ=1,NP
      PHTDEL(II,JJ)=PHIT(II,JJ)
8     V1TDEL(II,JJ)=V1(II,JJ)
      DO 9 II=1,NP
      DO 9 JJ=1,NUP
9     PSITDE(II,JJ)=PSIT(II,JJ)
      GO TO 300
10    DO 11 II=1,NP
      DO 11 JJ=1,NP
11    PHITAU(II,JJ)=PHIT(II,JJ)
      GO TO 300
12    DO 13 II=1,NP
      DO 13 JJ=1,NP
      PHTTD(II,JJ) = PHIT(II,JJ)
13    V1TTD(II,JJ) = V1(II,JJ)
      DO 14 II=1,NP
      DO 14 JJ=1,NUP
14    PSITTD(II,JJ)=PSIT(II,JJ)
300   CONTINUE
      TAU = XNTAU*T/XNT
      DELAY = XNDELA*T/XNT
      WRITE(6,15) TAU
15    FORMAT('0',20X,'TAU=(NTTAU*T)/NT=',F10.5)
      WRITE(6,16) DELAY
16    FORMAT('0',20X,'DELAY=(XNTTDE*T)/NT=',F10.5)
C     WRITE PHIT(T),PHIT(TAU),PHIT(T-DELAY)ANDPHIT(TAU-DELAY)
      WRITE(6,17)
17    FORMAT('OPHIT')
      DO 18 I=1,NP
18    WRITE(6,915)(PHIT(I,J),J=1,NP)
      WRITE(6,19)
19    FORMAT('OPHITAU')
      DO 20 I=1,NP
20    WRITE(6,915)(PHITAU(I,J),J=1,NP)
      WRITE(6,21)
21    FORMAT('OPHIT(T-DELAY)')
      DO 22 I=1,NP
22    WRITE(6,915)(PHTTD(I,J),J=1,NP)
      WRITE(6,23)
23    FORMAT('OPHIT(TAU-DELAY)')
      DO 24 I=1,NP
```

```
24      WRITE(6,915)(PHTATD(I,J),J=1,NP)
C       WRITE PS1T(T-DELAY),PS1T(TAU-DELAY)AND PS1T(DELAY)
        WRITE(6,252)
252     FORMAT('0PS1T(T)')
        DO 253 I = 1,NP
253     WRITE(6,915)(PS1T(I,J),J=1,NUP)
        WRITE(6,25)
25      FORMAT('0PS1T(T-DELAY)')
        DO 26 I=1,NP
26      WRITE(6,915)(PS1TTD(I,J),J=1,NUP)
        WRITE(6,27)
27      FORMAT('0PS1T(TAU-DELAY)')
        DO 28 I=1,NP
28      WRITE(6,915)(PS1TAT(I,J),J=1,NUP)
        WRITE(6,29)
29      FORMAT('0PS1T(DELAY)')
        DO 30 I=1,NP
30      WRITE(6,915)(PS1TDE(I,J),J=1,NUP)
C       CALCULATE AND WRITE PS2T(TAU-DELAY)AND PS2T(DELAY)
        DO 717 I = 1,NP
        DO 717 J = 1,NWP
        PS2TTD(I,J) = 0.0
        DO 717 K = 1,NP
717     PS2TTD(I,J) = PS2TTD(I,J) + V1TTD(I,K)*B2P(K,J)
        DO 718 I = 1,NP
        DO 718 J = 1,NWP
        PS2TDE(I,J) = 0.0
        DO 718 K = 1,NP
718     PS2TDE(I,J) = PS2TDE(I,J) + V1TDEL(I,K)*B2P(K,J)
        DO 719 I = 1,NP
        DO 719 J = 1,NWP
        PS2TAT(I,J) = 0.0
        DO 719 K = 1,NP
719     PS2TAT(I,J) = PS2TAT(I,J) + V1TATD(I,K)*B2P(K,J)
        WRITE(6,31)
31      FORMAT('0 PS2T(TAU-DELAY)')
        DO 32 I=1,NP
32      WRITE(6,915)(PS2TAT(I,J),J=1,NWP)
        WRITE(6,33)
33      FORMAT('0 PS2T(DELAY)')
        DO 34 I=1,NP
34        WRITE(6,915)(PS2TDE(I,J),J=1,NWP)
        WRITE(6,333)
333     FORMAT('0PS2(T-DELAY)')
        DO 334 I = 1,NP
334     WRITE(6,915)(PS2TTD(I,J),J=1,NWP)
CCCCC CALCULATE AND WRITE G(T,TAU)
        DO 704 I = 1,NP
        DO 704 J = 1,NWP
        PM(I,J) = 0.0
        DO 704 K = 1,NP
704     PM(I,J) = PM(I,J) + PHTATD(I,K)*PS2TDE(K,J)
        DO 36 I=1,NP
        DO 36 J=1,NWP
```

```
36    AM(I,J)=PM(I,J)+PS2TAT(I,J)
C     FIRST ROW
      DO 37 I=1,NP
      DO 37 J=1,NWP
      G(I,J)=PS2TTD(I,J)
      DO 37 K=1,NP
37    G(I,J)=G(I,J)+PHTTD(I,K)*PS2TDE(K,J)
C     SECOND ROW
      DO 38 I=1,NP
      N=NP+I
      DO 38 J=1,NWP
38    G(N,J)=0.0
C     THIRD ROW
      DO 39 I=1,NC
      N=NP+NP+I
      DO 39 J=1,NWP
39    G(N,J)=0.0
C     FOURTH ROW
      DO 801 I = 1,NC
      N = NP+NP+NC+I
      DO 801 J = 1,NWP
801   G(N,J) = 0.0
C     FIFTH ROW
      DO 41 I=1,NC
      N=NP+NP+NC+NC+I
      DO 41 J=1,NWP
      G(N,J)=0.0
      DO 41 K=1,NP
41    G(N,J)=G(N,J)+GCCP(I,K)*AM(K,J)
      NF=NP+NP+NC+NC+NC
      WRITE(6,42)
42    FORMAT ('0 G(T,TAU)')
      DO 43 I=1,NF
43    WRITE(6,915)(G(I,J),J=1,NWP)
CCCCC WRITE PL(T,TAU)
      DO 705 I = 1,NUP
      DO 705 J = 1,NWP
      PL(I,J) = 0.0
      DO 705 K = 1,NP
705   PL(I,J) = PL(I,J) + ECCP(I,K)*AM(K,J)
      WRITE(6,45)
45    FORMAT('0PL(T,TAU)')
      DO 46 I = 1,NUP
46    WRITE(6,915)(PL(I,J),J=1,NWP)
CCCCC CALCULATE AND WRITE F(T,TAU)
      DO 708 I = 1,NP
      DO 708 J = 1,NUP
      D(I,J) = 0.0
      DO 708 K = 1,NP
708   D(I,J) = D(I,J) + PHTTD(I,K)*PS1TDE(K,J)
CCCCC 1ST ROW
      DO 47 I = 1,NP
      DO 48 J = 1,NP
      F(I,J) = PHIT(I,J)
```

```
      DO 48 K = 1,NUP
48    F(I,J) = F(I,J)+PS1TTD(I,K)*ECCP(K,J)
      DO 49 J = 1,NP
      M = NP+J
      F(I,M) = 0.0
      DO 49 K = 1,NUP
49    F(I,M) = F(I,M)+D(I,K)*ECCP(K,J)
      DO 50 J = 1,NC
      M = NP+NP+J
      F(I,M) = DW(I,J)
      DO 50 K = 1,NUP
50    F(I,M) = F(I,M) + PS1TTD(I,K)*HC(K,J)
      DO 803 J = 1,NC
      M = NP+NP+NC+J
      F(I,M) = 0.0
      DO 803 K = 1,NUP
803   F(I,M) = F(I,M)+D(I,K)*HC(K,J)
      DO 52 J=1,NC
      M = NP+NP+NC+NC+J
52    F(I,M) = 0.0
47    CONTINUE
CCCCC 2ND ROW
      DO 53 I = 1,NP
      N = NP+I
      DO 54 J = 1,NP
54    F(N,J) = 1.
      DO 55 J = 1,NP
      M = NP+J
55    F(N,M) = 0.0
      DO 56 J = 1,NC
      M = NP+NP+J
56    F(N,M) = 0.0
      DO 58 J = 1,NC
      M = NP+NP+NC+J
58    F(N,M) = 0.0
      DO 804 J = 1,NC
      M = NP+NP+NC+NC+J
804   F(N,M) = 0.0
53    CONTINUE
CCCCC 3RD ROW
      DO 59 I = 1,NC
      N = NP+NP+I
      DO 60 J =1,NP
60    F(N,J) = GCCP(I,J)
      DO 61 J = 1,NP
      M = NP+J
61    F(N,M) = 0.0
      DO 62 J = 1,NC
      M = NP+NP+J
62    F(N,M) = FC(I,J)
      DO 64 J = 1,NC
      M = NP+NP+NC+J
64    F(N,M) = 0.0
      DO 805 J = 1,NC
```

```
      M = NP+NP+NC+NC+J
805   F(N,M) = 0.0
59    CONTINUE
CCCCC 4TH ROW
      DO 806 I = 1,NC
      N = NP+NP+NC+I
      DO 807 J = 1,NP
807   F(N,J) = 0.0
      DO 808 J = 1,NP
      M = NP+J
808   F(N,M) = 0.0
      DO 809 J = 1,NC
      M = NP+NP+J
809   F(N,M) = 1.
      DO 810 J = 1,NC
      M = NP+NP+NC+J
810   F(N,M) = 0.0
      DO 811 J = 1,NC
      M = NP+NP+NC+NC+J
811   F(N,M) = 0.0
806   CONTINUE
CCCCC FIFTH ROW
      DO 710 I = 1,NP
      DO 710 J = 1,NC
      DW(I,J) = 0.0
      DO 710 K = 1,NUP
710   DW(I,J) = DW(I,J) + PSITAT(I,K)*HC(K,J)
      DO 711 I = 1,NP
      DO 711 J = 1,NUP
      D(I,J) = 0.0
      DO 711 K = 1,NP
711   D(I,J) = D(I,J) + PHTATD(I,K)*PSITDE(K,J)
      DO 712 I = 1,NP
      DO 712 J = 1,NP
      AM(I,J) = 0.0
      DO 712 K = 1,NUP
712   AM(I,J) = AM(I,J) + PSITAT(I,K)*ECCP(K,J)
      DO 65 I = 1,NP
      DO 65 J = 1,NP
65    PT(I,J) = PHITAU(I,J)+AM(I,J)
      DO 713 I = 1,NP
      DO 713 J = 1,NC
      AM(I,J) = 0.0
      DO 713 K = 1,NUP
713   AM(I,J) = AM(I,J) + D(I,K)*HC(K,J)
      DO 714 I = 1,NP
      DO 714 J = 1,NP
      PM(I,J) = 0.0
      DO 714 K = 1,NUP
714   PM(I,J) = PM(I,J) + D(I,K)*ECCP(K,J)
      DO 72 I = 1,NC
      N = NP+NP+NC+NC+I
      DO 73 J = 1,NP
      F(N,J)=0.0
```

```
      DO 73 K = 1,NP
73    F(N,J) = F(N,J)+GCCP(I,K)*PT(K,J)
      DO 74 J = 1,NP
      M = NP+J
      F(N,M) = 0.0
      DO 74 K = 1,NP
74    F(N,M) = F(N,M)+GCCP(I,K)*PM(K,J)
      DO 812 J = 1,NC
      M = NP+NP+J
      F(N,M) = 0.0
      DO 812 K = 1,NP
812   F(N,M) = F(N,M) + GCCP(I,K)*DW(K,J)
      DO 75 J = 1,NC
      M = NP+NP+NC+J
      F(N,M) = 0.0
      DO 75 K = 1,NP
75    F(N,M) = F(N,M)+GCCP(I,K)*AM(K,J)
      DO 77 J = 1,NC
      M = NP+NP+NC+NC+J
77    F(N,M) = FC(I,J)
72    CONTINUE
CCCCC WRITE F(T,TAU)
      WRITE(6,78)
78    FORMAT('0F(T,TAU)')
      DO 79 I = 1,NF
79    WRITE(6,915)(F(I,J),J=1,NF)
CCCCC CALCULATE H1 AND H2
      DO 80 I = 1,NUP
      DO 81 J = 1,NP
81    H1(I,J) = ECCP(I,J)
      DO 82 J = 1,NP
      M = NP+J
82    H1(I,M) = 0.0
      DO 83 J = 1,NC
      M = NP+NP+J
83    H1(I,M) = HC(I,J)
      DO 85 J = 1,NC
      M = NP+NP+NC+J
85    H1(I,M) = 0.0
      DO 814 J = 1,NC
      M = NP+NP+NC+NC+J
814   H1(I,M) = 0.0
80    CONTINUE
      DO 86 I = 1,NUP
      DO 87 J = 1,NP
      H2(I,J) = 0.0
      DO 87 K = 1,NP
87    H2(I,J) = H2(I,J)+ECCP(I,K)*PT(K,J)
      DO 88 J = 1,NP
      M = NP+J
      H2(I,M) = 0.0
      DO 88 K = 1,NP
88    H2(I,M) = H2(I,M)+ECCP(I,K)*PM(K,J)
      DO 89 J = 1,NC
```

```
      M = NP+NP+J
      H2(I,M) = 0.0
      DO 89 K = 1,NP
89    H2(I,M) = H2(I,M)+ECCP(I,K)*DW(K,J)
      DO 813 J = 1,NC
      M = NP+NP+NC+J
      H2(I,M) = 0.0
      DO 813 K = 1,NP
813   H2(I,M) = H2(I,M)+ECCP(I,K)*AM(K,J)
      DO 91 J = 1,NC
      M = NP+NP+NC+J
91    H2(I,M) = HC(I,J)
86    CONTINUE
CCCCC CALCULATE HA,HB AND PLB
      DO 92 I = 1,NUP
      DO 92 J = 1,NF
92    HA(I,J) = H1(I,J)-H2(I,J)
      DO 715 I = 1,NUP
      DO 715 J = 1,NF
      D(I,J) = 0.0
      DO 715 K = 1,NF
715   D(I,J) = D(I,J) + H1(I,K)*F(K,J)
      DO 93 I = 1,NUP
      DO 93 J = 1,NF
93    HB(I,J) = D(I,J)-H2(I,J)
      DO 716 I = 1,NUP
      DO 716 J = 1,NWP
      D(I,J) = 0.0
      DO 716 K = 1,NF
716   D(I,J) = D(I,J) + H1(I,K)*G(K,J)
      DO 94 I = 1,NUP
      DO 94 J = 1,NWP
94    PLB(I,J) = D(I,J)-PL(I,J)
CCCCC CALCULATE PXSS AND WRITE
      IT = 2
      IMAX = 30
      CALL MMMT(G,W,GWG,NF,NWP,NFM,NWPM,NWPM,NFM,D,NFM)
      CALL MODCAL(F,GWG,PXSS,AM,PM,NF,NFM,IMAX,IT)
      WRITE(6,95)
95    FORMAT('0STEADY-STATE COVARIANCE OF STATES')
      DO 96 I = 1,NF
96    WRITE(6,915)(PXSS(I,J),J=1,NF)
CCCCC CALCULATE PEAXSS
      CALL MMMT(HA,PXSS,HPH,NUP,NF,NUPM,NFM,NFM,NUPM,D,NFM)
      CALL MMMT(PLB,W,PLWPL,NUP,NWP,NUPM,NWPM,NWPM,NUPM,
     1 D,NFM)
      DO 97 I = 1,NUP
      DO 97 J = 1,NUP
97    PEAXSS(I,J) = HPH(I,J)+PLWPL(I,J)
      WRITE(6,98)
98    FORMAT('0STEADY-STATE COVARIANCE OF E1')
      DO 99 I = 1,NUP
99    WRITE(6,915)(PEAXSS(I,J),J=1,NUP)
CCCCC CALCULATE AND WRITE PEBXSS
```

```
      CALL MMMT(HB.PXSS.HPH.NUP.NF.NUPM.NFM.NFM.NUPM.D.NFM)
      CALL MMMT(PLB.W.PLWPL.NUP.NWP.NUPM.NWPM.NWPM.NUPM.
     1 D.NFM)
      DO 100 I = 1.NUP
      DO 100 J = 1.NUP
100   PEBXSS(I.J) = HPH(I.J)+PLWPL(I.J)
      WRITE(6.101)
101   FORMAT('OSTEADY-STATE COVARIANCE OF E2')
      DO 102 I = 1.NUP
102   WRITE(6.915)(PEBXSS(I.J).J=1.NUP)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCC CALCULATE X(TIME) TO A UNIT-STEP INPUT FROM ZERO
CCCCC INITIAL CONDITIONS CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6.274) NTIME.NWRITE
274   FORMAT('OUNIT-STEP TIME RESPONSE FOR TAU POINT'/
     1 ' NO. OF T-INCREMENTED POINTS IN THE UNIT-STEP TIME
     2 RESPONSE = '.I5/' TIME RESPONSE TO BE WRITTEN
     3 EVERY '.I5.'*T SECONDS')
      TIME = 0.0
      DO 200 I = 1.NUP
200   YC2(I) = 0.0
      WRITE(6.201) TIME.(YC2(I).I=1.NUP)
201   FORMAT('OTIME='.G13.6.'YC2(TIME-T+TAU+DELAY) = '.
     2 G13.6)
      DO 202 I = 1.NUP
202   E1(I) = 0.0
      WRITE(6.203)(E1(I).I=1.NUP)
203   FORMAT(10X.'E1 = '.G13.6)
      WRITE(6.204)
204   FORMAT('0')
      DO 205 I = 1.NF
205   X(I) = 0.0
      WRITE(6.206) TIME.(X(I).I=1.NF)
206   FORMAT('OTIME='.G13.6.'X='.7G13.6./.25X.7G13.6)
CCCCC CALCULATE YC1(TIME+DELAY)
      DO 207 I = 1.NUP
      Q = 0.0
      DO 208 J = 1.NF
208   Q = Q + H1(I.J)*X(J)
207   YC1(I) = Q
      WRITE(6.209)(YC1(I).I=1.NUP)
209   FORMAT(10X.'YC1(TIME+DELAY) = '.G13.6)
CCCCC CALCULATE E2
      DO 210 I = 1.NUP
210   E2(I) = YC1(I)-YC2(I)
      WRITE(6.211)(E2(I).I=1.NUP)
211   FORMAT(10X.'E2 = '.G13.6)
CCCCC CALCULATE YC2(TIME+TAU+DELAY)
      DO 217 I = 1.NUP
217   YC2(I) = 0.0
      DO 213 I = 1.NUP
```

```
      Q = 0.0
      DO 214 J = 1,NF
214   Q = Q + H2(I,J)*X(J)
213   YC2(I) = YC2(I) + Q
      DO 215 I = 1,NUP
      Q = 0.0
      DO 216 J = 1,NWP
216   Q = Q+PL(I,J)
215   YC2(I) = YC2(I) + Q
      WRITE(6,218)(YC2(I),I=1,NUP)
218   FORMAT(10X,'YC2(TIME+TAU+DELAY) =',G13.6)
CCCCC CALCULATE E1
      DO 219 I = 1,NUP
219   E1(I) = YC1(I)-YC2(I)
      WRITE(6,203)(E1(I),I=1,NUP)
      IWRITE=0
      THETA = 0.0
      DO 220 JT = 1,NTIME
      DO 221 I = 1,NF
      Q = 0.0
      DO 222 J = 1,NF
222   Q = Q+F(I,J)*X(J)
221   XW(I) = Q
      DO 223 I = 1,NF
      Q = 0.0
      DO 224 J = 1,NWP
224   Q = Q+G(I,J)
223   X(I) = XW(I)+Q
      TIME = TIME+T
CCCCC CALCULATE PITCH ANGLE(TIME+T/2.0)
      THETA = THETA+X(2)*T
CCCCC CALCULATE YC1(TIME)
      DO 225 I = 1,NUP
      Q = 0.0
      DO 226 J = 1,NF
226   Q = Q+H1(I,J)*X(J)
225   YC1(I) = Q
CCCCC CALCULATE E2
      DO 227 I = 1,NUP
227   E2(I) = YC1(I)-YC2(I)
CCCCC CALCULATE YC2(TIME+TAU+DELAY)
      DO 228 I = 1,NUP
228   YC2(I) = 0.0
      DO 229 I = 1,NUP
      Q = 0.0
      DO 230 J = 1,NF
230   Q = Q+H2(I,J)*X(J)
229   YC2(I) = YC2(I)+Q
      DO 231 I = 1,NUP
      Q = 0.0
      DO 232 J = 1,NWP
232   Q = Q+PL(I,J)
231   YC2(I) = YC2(I) + Q
CCCCC CALCULATE E1
```

```
      DO 245 I = 1,NUP
245   E1(I) = YC1(I)-YC2(I)
      IWRITE = IWRITE+1
      IF(IWRITE.NE.NWRITE) GO TO 220
      IWRITE = 0
      WRITE(6,206) TIME,(X(I),I=1,NF)
      WRITE(6,209) (YC1(I),I=1,NUP)
      WRITE(6,211)(E2(I),I=1,NUP)
      WRITE(6,218)(YC2(I),I=1,NUP)
      WRITE(6,203)(E1(I),I=1,NUP)
      WRITE(6,124) THETA
124   FORMAT('     PITCH ANGLE(TIME+T/2.0) = ',G13.6)
220   CONTINUE
      STOP
      END
```

```
     A7 MODEL --- 4TH ORDER PLANT, 1ST ORDER CONTROLLER

NO. OF PLANT STATES =   4
NO. OF PLANT INPUTS =    1
NO. OF DISTURBANCE INPUTS =  1
NO. OF PLANT OUTPUTS =    3
NO. OF CONTROLLER STATES (EACH CONTROLLER) =    1

PLANT STATE MATRIX -- AP
 -.757200      1.00000      .0           -.109000
 -3.70120     -.551100      .0           -6.07340
   .0          .0         -2000.00         .0
   .0          .0           .0           -20.0000

PLANT CONTROL INPUT MATRIX -- B1P
   .0
   .0
   .0
  20.0000

PLANT DISTURBANCE INPUT MATRIX -- B2P
   .0
   .0
  2000.00
   .0

PLANT OUTPUT MATRIX -- CP
   .0          1.00000      .0             .0
  7.07074     -.119810      .0           -.186760
   .0         .0           1.00000         .0

CONTROLLER STATE MATRIX -- FC
  .951220

CONTROLLER CONTROL INPUT MATRIX -- GC
  .0          .475907E-01 -.142772E-01

CONTROLLER OUTPUT MATRIX (STATES) -- HC
  .174533E-01

CONTROLLER OUTPUT MATRIX (INPUTS) -- EC
  .250000     .425690E-03 -.127707E-03
```

```
T =       0.0125
NT =      50
NDELAY=   30
NTAU =    40
T = SAMPLE RATE.
NT = NB. OF EVENLY-SPACED SUBINTERVALS INTO WHICH T IS DIVIDED.
DELTA = T/NT = INCREMENT USED IN THE NUMERICAL INTEGRATIONS.


DISTURBANCE COVARIANCE MATRIX -- W
  1.00000
   1 1600  800

                    TAU=(NTTAU*T)/NT=     0.01000

                    DELAY=(XNTTDE*T)/NT=     0.00750

PHIT
 .990389       .123972E-01   .0             -.163455E-02
-.458838E-01   .992886       .0             -.669037E-01
 .0            .0            .139001E-10     .0
 .0            .0            .0              .779015

PHITAU
 .992255       .993384E-02   .0             -.126714E-02
-.367667E-01   .994377       .0             -.548705E-01
 .0            .0            .206190E-08     .0
 .0            .0            .0              .818824

PHIT(T-DELAY)
 .996166       .498339E-02   .0             -.590915E-03
-.184443E-01   .997231       .0             -.288528E-01
 .0            .0            .454072E-04     .0
 .0            .0            .0              .904885

PHIT(TAU-DELAY)
 .998093       .249580E-02   .0             -.284189E-03
-.923735E-02   .998626       .0             -.147986E-01
 .0            .0            .673842E-02     .0
 .0            .0            .0              .951255
```

```
PS1T(T)
-.193525E-03
-.872368E-02
 .0
 .221245
PS1T(T-DELAY)
-.287953E-04
-.146747E-02
 .0
 .951662E-01
PS1T(TAU-DELAY)
-.700829E-05
-.373146E-03
 .0
 .487715E-01
PS1T(DELAY)
-.664555E-04
-.324668E-02
 .0
 .139297
PS2T(TAU-DELAY)
 .0
 .0
 1.01388
 .0
PS2T(DELAY)
 .0
 .0
 1.02075
 .0
PS2(T-DELAY)
 .0
 .0
 1.02071
 .0
```

G(T,TAU)
.0
.0
1.02075
.0
.0
.0
.0
.0
.0
.0
-.145735E-01
PL(T,TAU)
-.130358E-03

F(T,TAU)

.990389	.123900E-01	.367736E-08	-.163455E-02	-.495716E-06	-.411648E-04	.210324E-07
.130934E-07	-.502573E-06	-.287443E-05	.0			
-.458882E-01	.992519	.187406E-06	-.669035E-01	-.218388E-04	-.181352E-02	.926586E-06
.576831E-06	-.256122E-04	-.126634E-03	.0			
.0	.0	.139001E-10	.0	.0	.0	.0
.0	.0	.0	.0			
.286445E-03	.237867E-01	-.121534E-04	.779007	.379396E-03	.315055E-01	-.160972E-04
-.100210E-04	.166096E-02	.219995E-02	.0			
1.00000	1.00000	1.00000	1.00000	.0	.0	.0
.0	.0	.0	.0			
1.00000	1.00000	1.00000	1.00000	.0	.0	.0
.0	.0	.0	.0			
1.00000	1.00000	1.00000	1.00000	.0	.0	.0
.0	.0	.0	.0			
1.00000	1.00000	1.00000	1.00000	.0	.0	.0
.0	.0	.0	.0			
.336501	-.570184E-02	-.142772E-01	-.888804E-02	.0	.0	.0
.0	.951220	.0	.0			
.0	.0	.0	.0	.0	.0	.0
.0	1.00000	.0	.0			
.334103	-.243544E-02	.553588E-07	-.739123E-02	-.356936E-05	-.296403E-03	.151442E-06
.942777E-07	-.756973E-05	-.206971E-04	.951220			

```
     ITER=10

STEADY-STATE COVARIANCE OF STATES
 .291637E-01  .231938E-01  .426071E-17  .119365E-01  .657724E-01  .657724E-01  .657724E-01
 .657724E-01  .184638      .184052      .184899
 .231938E-01  .208859      .217135E-15 -.329128E-01  .179477      .179477      .179477
 .179477      .827483E-01  .804430E-01  .844288E-01
 .426071E-17  .217135E-15  83.3546     -.140813E-13  .115863E-08  .115863E-08  .115863E-08
 .115863E-08 -.165420E-10 -.229935E-21 -1.19007
 .119365E-01 -.329128E-01 -.140813E-13  .236479      .186073      .186073      .186073
 .186073     -.775758E-01 -.843710E-01 -.723463E-01
 .657724E-01  .179477      .115863E-08  .186073      83.8335      83.8335      83.8335
 83.8335     -.990935      .189809     -.925378
 .657724E-01  .179477      .115863E-08  .186073      83.8335      83.8335      83.8335
 83.8335     -.990935      .189809     -.925378
 .657724E-01  .179477      .115863E-08  .186073      83.8335      83.8335      83.8335
 83.8335     -.990935      .189809     -.925378
 .657724E-01  .179477      .115863E-08  .186073      83.8335      83.8335      83.8335
 83.8335     -.990935      .189809     -.925378
 .184638      .827483E-01 -.165420E-10 -.775758E-01 -.990935     -.990935     -.990935
-.990935      1.45795      1.44918      1.44919
 .184052      .804430E-01 -.229935E-21 -.843710E-01  .189809      .189809      .189809
 .189809      1.44918      1.45796      1.44081
 .184899      .844288E-01 -1.19007     -.723463E-01 -.925378     -.925378     -.925378
-.925378      1.44919      1.44081      1.45780

STEADY-STATE COVARIANCE OF E1
 .594630E-04

STEADY-STATE COVARIANCE OF E2
 .380986E-05
```

```
UNIT-STEP TIME RESPONSE FOR TAU POINT
NO. OF T-INCREMENTED POINTS IN THE UNIT-STEP TIME RESPONSE =  1600
TIME RESPONSE TO BE WRITTEN EVERY    800*T SECONDS

TIME=  .0          YC2(TIME-T+TAU+DELAY) =     .0
          E1 =    .0

TIME=  .0          X=  .0          .0          .0          .0          .0          .0          .0
                          .0          .0          .0          .0
          YC1(TIME+DELAY) =    .0
          E2 =    .0
          YC2(TIME+TAU+DELAY) = -.130358E-03
          E1 =    .130358E-03

TIME=  9.99986     X= -.146612     -.992159E-01  1.02075     .909194E-01  .865843     .865343     .8658
                          .865843     -1.31510     -1.31510     -1.31630
          YC1(TIME+DELAY) =  -.483307E-01
          E2 =    .366420E-04
          YC2(TIME+TAU+DELAY) = -.483673E-01
          E1 =    .366420E-04
   PITCH ANGLE(TIME+T/2.0) =   -1.09443

TIME=  19.9988     X= -.146612     -.992159E-01  1.02075     .909194E-01  .865843     .865843     .8658
                          .865843     -1.31510     -1.31510     -1.31630
          YC1(TIME+DELAY) =  -.483307E-01
          E2 =    .366420E-04
          YC2(TIME+TAU+DELAY) = -.483673E-01
          E1 =    .366420E-04
   PITCH ANGLE(TIME+T/2.0) =   -2.08625
```

THE VARIATION C

LISTING

AND

EXAMPLES (CASE I AND CASE II)

```
      COMMON ELEMX,MAXI,MAXJ
      DIMENSION AP(4,4),B1P(4,2),B2P(4,2),CP(3,4),FC(2,2),
     1 HC(2,2),EC(2,3),PHIT(10,10),PS1T(10,10),F(14,14),
     2 DELPHI(10,10),EIP(10,4),EIV(10,4),FH(10,10),
     3 KWA(10),RF(20),RR(20),ID(20),ECCP(2,4),GCCP(2,4),
     4 INDEX(10),W(2,2),PS(10,10),PS1TAT(10,10),GC(2,3),
     5 PS1TTD(10,10),PHTATD(10,10),PHTDEL(10,10),AH(10,10),
     6 PHTTD(10,10),V1(4,4),V1TDEL(4,4),V1TATD(4,4),
     7 PS2TDE(10,10),PM(14,14),PS2TTD(10,10),V1TTD(4,4),
     8 AM(14,14),G(14,2),D(14,14),DW(14,14),PL(2,2),
     9 H1(2,14),H2(2,14),HA(2,14),HB(2,14),PLB(2,2),
     A PLWPL(2,2),PEAXSS(2,2),PEBXSS(2,2),PXSS(14,14),
     B FHST(10,10),DD(10),PT(10,10),PS1TDE(10,10),
     C PHITAU(10,10),PS2TAT(10,10),GWG(14,14),
     D X(14),XW(14),E1(2),E2(2),EA(2),E11(2),E22(2),
     E YC1(2),YC2(2),PS1TAU(10,10),V1TAU(4,4),HPH(2,2)
     F PS2TAU(10,10)
CCCCC PROVIDE MAXIMA FOR CALLED ARRAYS
      NPM = 4
      NUPM = 2
      NWPM = 2
      NOPM = 3
      NCM = 2
      NHM = 2*NPM + NUPM
      NFM = 2*NPM + 3*NCM
      NRRM = 4*NPM + 4
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCC READ INPUT DATA CCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,2222)
2222  FORMAT('1')
      READ(5,900) ID
  900 FORMAT(20A4)
      WRITE(6,902) ID
  902 FORMAT('1',20A4)
      READ(5,906)NP,NUP,NWP,NOP,NC
  906 FORMAT(5I3)
      WRITE(6,908) NP,NUP,NWP,NOP,NC
908   FORMAT('0NO. OF PLANT STATES =',I3/
     1 ' NO. OF PLANT INPUTS = ',I3/
     2 ' NO. OF DISTURBANCE INPUTS = ', I3/
     4 ' NO. OF PLANT OUTPUTS = ', I3/
     5 ' NO. OF CONTROLLER STATES (EACH CONTROLLER) = ',I3)
       WRITE(6,910)
  910 FORMAT('0PLANT STATE MATRIX -- AP')
  110 DO 112 I = 1,NP
      READ(5,914) (AP(I,J),J=1,NP)
112   WRITE(6,913) (AP(I,J),J=1,NP)
913   FORMAT(' ',8G13.6)
914   FORMAT(4F13.7)
915   FORMAT(7G13.6)
      WRITE(6,916)
```

```
  916 FORMAT('OPLANT CONTROL INPUT MATRIX -- B1P')
  120 DO 122 I = 1,NP
      READ(5,914)(B1P(I,J),J=1,NUP)
122   WRITE(6,913)(B1P(I,J),J=1,NUP)
      WRITE(6,918)
  918 FORMAT('OPLANT DISTURBANCE INPUT MATRIX -- B2P')
  130 DO 132 I=1,NP
      READ(5,914)(B2P(I,J),J=1,NWP)
132   WRITE(6,913)(B2P(I,J),J=1,NWP)
      WRITE(6,920)
  920 FORMAT('OPLANT OUTPUT MATRIX -- CP')
  140 DO 142 I=1,NOP
      READ(5,914)(CP(I,J),J=1,NP)
142   WRITE(6,913)(CP(I,J),J=1,NP)
      WRITE(6,922)
  922 FORMAT('OCONTROLLER STATE MATRIX -- FC')
  150 DO 152 I =1,NC
      READ(5,914)(FC(I,J),J=1,NC)
152   WRITE(6,913)(FC(I,J),J=1,NC)
      WRITE(6,924)
  924 FORMAT('OCONTROLLER CONTROL INPUT MATRIX -- GC ')
  160 DO 162 I=1,NC
      READ(5,914)(GC(I,J),J=1,NOP)
  162 WRITE(6,913)(GC(I,J),J=1,NOP)
      WRITE(6,925)
  925 FORMAT('OCONTROLLER OUTPUT MATRIX (STATES) -- HC')
  170 DO 172 I=1,NUP
      READ(5,914)(HC(I,J),J=1,NC)
  172 WRITE(6,913)(HC(I,J),J=1,NC)
      WRITE(6,926)
926   FORMAT('OCONTROLLER OUTPUT MATRIX (INPUTS) -- EC')
  180 DO 182 I=1,NUP
      READ(5,914)(EC(I,J),J=1,NOP)
  182 WRITE(6,913)(EC(I,J),J=1,NOP)
      READ(5,928)T,NT,NDELAY,NTAU
928   FORMAT(F10.4,5I5)
      XNT=NT
      XNTAU = NTAU
      XNDELA = NDELAY
      NIFT = NT-NDELAY
      DELTA=T/XNT
      WRITE(6,930)T,NT,NDELAY,NTAU
930   FORMAT('1T = ',F10.4/
     1 ' NT = ',I5/
     2 ' NDELAY= ',I3//
     3 ' NTAU = ',I3//
     4 ' T = SAMPLE RATE.'/
     5 ' NT = NB. OF EVENLY-SPACED SUBINTERVALS INTO'/
     6 '        WHICH T IS DIVIDED.'//
     7 ' DELTA = T/NI/NT = INCREMENT USED IN THE'/
     8 '             NUMERICAL INTEGRATIONS.'//
      WRITE(6,931)
  931 FORMAT('ODISTURBANCE COVARIANCE MATRIX -- W')
  184 DO 186 I=1,NWP
```

```
      READ(5,914)(W(I,J),J=1,NWP)
  186 WRITE(6,913)(W(I,J),J=1,NWP)
      DO 850 I = 1,NWP
      DO 850 J = 1,NWP
850   W(I,J) = W(I,J)/T
      READ(5,888)IXTIME,NXTIME,NXWRIT
888   FORMAT(3I5)
      WRITE(6,988) IXTIME,NXTIME,NXWRIT
988   FORMAT(3I5)
      NII=NI-1
      NWRITE=NXWRIT
      NTIME=NXTIME
      ITIME=IXTIME
CCCCC CALCULATE ECCP AND GCCP  CCCCCCCCCCCCCCCCCCCCCCCCCCC
      DO 701 I = 1,NUP
      DO 701 J = 1,NP
      ECCP(I,J) = 0.0
      DO 701 K = 1,NOP
701   ECCP(I,J) = ECCP(I,J) + EC(I,K)*CP(K,J)
      DO 702 I = 1,NC
      DO 702 J = 1,NP
      GCCP(I,J) = 0.0
      DO 702 K = 1,NOP
702   GCCP(I,J) = GCCP(I,J) + GC(I,K)*CP(K,J)
      DELHLF=DELTA/2.0
      TI=0
      DO 1  I=1,NP
      DO 1  J=1,NP
      V1(I,J)=0.0
      PT(I,J)=0.0
1     PS(I,J)=0.0
      IDEL=0
      DO 2  I=1,NP
2     V1(I,I) = DELHLF
      IMPULS = 0
      IPRINT=9
      DAPPRX=0.0
      EPS = 1.0E-7
      NTERMS=6
      IPOLE=0
      WMAX=-1.0
      DO 300 I = 1,NT
      CALL MADD(V1,PT,V1,NP,NPM,NHM,NPM)
      TI=TI+DELTA
      CALL EXPK2(AP,AH,B1P,DAPPRX,DD,DELPHI,EIF,EIV,EPS,
     1 PHIT,FH,FHST,PSIT,IMPULS,IPOLE,IPRINT,KWA,NHM,
     2 NRRM,NTERMS,NUP,NUPM,NP,NPM,RF,RR,TI,WMAX,INDEX)
      DO 3 II=1,NP
      DO 3 JJ=1,NP
3     PT(II,JJ)=PHIT(II,JJ)
      CALL MBYCON(DELHLF,PT,NP,NHM)
      CALL MADD(V1,PT,V1,NP,NPM,NHM,NPM)
      IDEL=IDEL+1
      IF(IDEL.EQ.NDELAY) GO TO 7
```

```
      IF(IDEL.EQ.NTAU) GO TO 10
      IF(IDEL.EQ.NIFT)GO TO 12
      GO TO 300
7     DO 8 II=1,NP
      DO 8 JJ=1,NP
      PHTDEL(II,JJ)=PHIT(II,JJ)
8     V1TDEL(II,JJ)=V1(II,JJ)
      DO 9 II=1,NP
      DO 9 JJ=1,NUP
9     PSITDE(II,JJ)=PSIT(II,JJ)
      GO TO 300
10    DO 11 II=1,NP
      DO 11 JJ=1,NP
      PHITAU(II,JJ) = PHIT(II,JJ)
11    V1TAU(II,JJ) = V1(II,JJ)
      DO 505 II = 1,NP
      DO 505 JJ = 1,NUP
505   PSITAU(II,JJ) = PSIT(II,JJ)
      GO TO 300
12    DO 13 II=1,NP
      DO 13 JJ=1,NP
      PHTTD(II,JJ) = PHIT(II,JJ)
13    V1TTD(II,JJ) = V1(II,JJ)
      DO 14 II=1,NP
      DO 14 JJ=1,NUP
14    PSITTD(II,JJ)=PSIT(II,JJ)
300   CONTINUE
      TAU=XNTAU*T/XNT
      DELAY=XNDELA*T/XNT
      WRITE(6,15) TAU
15    FORMAT('0',20X,'TAU=(NTAU*T)/NT=',F10.5)
      WRITE(6,16) DELAY
16    FORMAT('0',20X,'DELAY=(XNDELA*T)/NT=',F10.5)
C-----WRITE PHIT(T),PHIT(TAU),PHIT(T-DELAY)
      WRITE(6,17)
17    FORMAT('0PHIT')
      DO 18 I=1,NP
18    WRITE(6,915)(PHIT(I,J),J=1,NP)
      WRITE(6,19)
19    FORMAT('0PHITAU')
      DO 20 I=1,NP
20    WRITE(6,915)(PHITAU(I,J),J=1,NP)
      WRITE(6,21)
21    FORMAT('0PHIT(T-DELAY)')
      DO 22 I=1,NP
22    WRITE(6,915)(PHTTD(I,J),J=1,NP)
C-----WRITE PSIT(T-DELAY),PSIT(TAU) AND PSIT(DELAY)
      WRITE(6,252)
252   FORMAT('0PSIT(T)')
      DO 253 I = 1,NP
253   WRITE(6,915)(PSIT(I,J),J=1,NUP)
      WRITE(6,25)
25    FORMAT('0PSIT(T-DELAY)')
      DO 26 I=1,NP
```

```
26      WRITE(6,915)(PS1TTD(I,J),J=1,NUP)
        WRITE(6,27)
27      FORMAT('0PS1T(TAU)')
        DO 28 I=1,NP
28      WRITE(6,915)(PS1TAU(I,J),J=1,NUP)
        WRITE(6,29)
29      FORMAT('0PS1T(DELAY)')
        DO 30 I=1,NP
30      WRITE(6,915)(PS1TDE(I,J),J=1,NUP)
C-----CALCULATE AND WRITE PS2T(TAU),PS2T(DELAY) AND PS2T(T-DELAY)
        DO 717 I = 1,NP
        DO 717 J = 1,NWP
        PS2TTD(I,J) = 0.0
        DO 717 K = 1,NP
717     PS2TTD(I,J) = PS2TTD(I,J) + V1TTD(I,K)*B2P(K,J)
        DO 718 I = 1,NP
        DO 718 J = 1,NWP
        PS2TDE(I,J) = 0.0
        DO 718 K = 1,NP
718     PS2TDE(I,J) = PS2TDE(I,J) + V1TDEL(I,K)*B2P(K,J)
        DO 719 I = 1,NP
        DO 719 J = 1,NWP
        PS2TAU(I,J) = 0.0
        DO 719 K = 1,NP
719     PS2TAU(I,J) = PS2TAU(I,J) + V1TAU(I,K)*B2P(K,J)
        WRITE(6,31)
31      FORMAT('0PS2T(TAU)')
        DO 32 I=1,NP
32      WRITE(6,915)(PS2TAU(I,J),J=1,NWP)
        WRITE(6,33)
33      FORMAT('0 PS2T(DELAY)')
        DO 34 I=1,NP
34        WRITE(6,915)(PS2TDE(I,J),J=1,NWP)
        WRITE(6,333)
333     FORMAT('0PS2(T-DELAY)')
        DO 334 I = 1,NP
334     WRITE(6,915)(PS2TTD(I,J),J=1,NWP)
CCCCC CALCULATE AND WRITE G(T,TAU)
C       FIRST ROW
        DO 37 I=1,NP
        DO 37 J=1,NWP
        G(I,J)=PS2TTD(I,J)
        DO 37 K=1,NP
37      G(I,J)=G(I,J)+PHTTD(I,K)*PS2TDE(K,J)
C       SECOND ROW
        DO 38 I=1,NP
        N=NP+I
        DO 38 J=1,NWP
38      G(N,J)=0.0
C       THIRD ROW
        DO 39 I=1,NC
        N=NP+NP+I
        DO 39 J=1,NWP
39      G(N,J)=0.0
```

```
C       FOURTH ROW
        DO 801 I = 1,NC
        N = NP+NP+NC+I
        DO 801 J = 1,NWP
801     G(N,J) = 0.0
C       FIFTH ROW
        DO 41 I=1,NC
        N = NP+NP+NC+NC+I
        DO 41 J=1,NWP
        G(N,J)=0.0
        DO 41 K=1,NP
41      G(N,J) = G(N,J)+GCCP(I,K)*PS2TAU(K,J)
        NF = NP+NP+NC+NC+NC
        WRITE(6,42)
42      FORMAT ('0 G(T,TAU)')
        DO 43 I=1,NF
43      WRITE(6,915)(G(I,J),J=1,NWP)
CCCCC WRITE PL(T,TAU)
        DO 705 I = 1,NUP
        DO 705 J = 1,NWP
        PL(I,J) = 0.0
        DO 705 K = 1,NP
705     PL(I,J) = PL(I,J) + ECCP(I,K)*PS2TAU(K,J)
        WRITE(6,45)
45      FORMAT('0PL(T,TAU)')
        DO 46 I = 1,NUP
46      WRITE(6,915)(PL(I,J),J=1,NWP)
CCCCC CALCULATE AND WRITE F(T,TAU)
        DO 708 I = 1,NP
        DO 708 J = 1,NUP
        D(I,J) = 0.0
        DO 708 K = 1,NP
708     D(I,J) = D(I,J) + PHTTD(I,K)*PS1TDE(K,J)
CCCCC 1ST ROW
        DO 47 I = 1,NP
        DO 48 J = 1,NP
        F(I,J) = PHIT(I,J)
        DO 48 K = 1,NUP
48      F(I,J) = F(I,J)+PS1TTD(I,K)*ECCP(K,J)
        DO 49 J = 1,NP
        M = NP+J
        F(I,M) = 0.0
        DO 49 K = 1,NUP
49      F(I,M) = F(I,M)+D(I,K)*ECCP(K,J)
        DO 802 J = 1,NC
        M = NP+NP+J
        F(I,M) = 0.0
        DO 802 K = 1,NUP
802     F(I,M) = F(I,M)+PS1TTD(I,K)*HC(K,J)
        DO 50 J = 1,NC
        M = NP+NP+NC+J
        F(I,M) = 0.0
        DO 50 K = 1,NUP
50      F(I,M) = F(I,M)+D(I,K)*HC(K,J)
```

```
      DO 52 J=1,NC
      M = NP+NP+NC+NC+J
52    F(I,M) = 0.0
47    CONTINUE
CCCCC 2ND ROW
      DO 53 I = 1,NP
      N = NP+I
      DO 54 J = 1,NP
54    F(N,J) = 1.
      DO 55 J = 1,NP
      M = NP+J
55    F(N,M) = 0.0
      DO 56 J = 1,NC
      M = NP+NP+J
56    F(N,M) = 0.0
      DO 58 J = 1,NC
      M = NP+NP+NC+J
58    F(N,M) = 0.0
      DO 803 J = 1,NC
      M = NP+NP+NC+NC+J
803   F(N,M) = 0.0
53    CONTINUE
CCCCC 3RD ROW
      DO 59 I = 1,NC
      N = NP+NP+I
      DO 60 J =1,NP
60    F(N,J) = GCCP(I,J)
      DO 61 J = 1,NP
      M = NP+J
61    F(N,M) = 0.0
      DO 62 J = 1,NC
      M = NP+NP+J
62    F(N,M) = FC(I,J)
      DO 64 J = 1,NC
      M = NP+NP+NC+J
64    F(N,M) = 0.0
      DO 804 J = 1,NC
      M = NP+NP+NC+NC+J
804   F(N,M) = 0.0
59    CONTINUE
CCCCC 4TH ROW
      DO 805 I = 1,NC
      N = NP+NP+NC+I
      DO 806 J = 1,NP
806   F(N,J) = 0.0
      DO 807 J = 1,NP
      M = NP+J
807   F(N,M) = 0.0
      DO 808 J = 1,NC
      M = NP+NP+J
808   F(N,M) = 1.
      DO 809 J = 1,NC
      M = NP+NP+NC+J
809   F(N,M) = 0.0
```

```
      DO 810 J = 1,NC
      M = NP+NP+NC+NC+J
810   F(N,M) = 0.0
805   CONTINUE
      DO 710 I = 1,NP
      DO 710 J = 1,NC
      AM(I,J) = 0.0
      DO 710 K = 1,NUP
710   AM(I,J) = AM(I,J) + PSITAU(I,K)*HC(K,J)
      DO 714 I = 1,NP
      DO 714 J = 1,NP
      PM(I,J) = 0.0
      DO 714 K = 1,NUP
714   PM(I,J) = PM(I,J) + PSITAU(I,K)*ECCP(K,J)
      DO 72 I = 1,NC
      N = NP+NP+NC+NC+I
      DO 73 J = 1,NP
      F(N,J)=0.0
      DO 73 K = 1,NP
73    F(N,J) = F(N,J)+GCCP(I,K)*PHITAU(K,J)
      DO 74 J = 1,NP
      M = NP+J
      F(N,M) = 0.0
      DO 74 K = 1,NP
74    F(N,M) = F(N,M)+GCCP(I,K)*PM(K,J)
      DO 811 J = 1,NC
      M = NP+NP+J
811   F(N,M) = 0.0
      DO 75 J = 1,NC
      M = NP+NP+NC+J
      F(N,M) = 0.0
      DO 75 K = 1,NP
75    F(N,M) = F(N,M)+GCCP(I,K)*AM(K,J)
      DO 77 J = 1,NC
      M = NP+NP+NC+NC+J
77    F(N,M) = FC(I,J)
72    CONTINUE
CCCCC WRITE F(T,TAU)
      WRITE(6,78)
78    FORMAT('OF(T,TAU)')
      DO 79 I = 1,NF
79    WRITE(6,915)(F(I,J),J=1,NF)
CCCCC CALCULATE H1 AND H2
      DO 80 I = 1,NUP
      DO 81 J = 1,NP
81    H1(I,J) = ECCP(I,J)
      DO 82 J = 1,NP
      M = NP+J
82    H1(I,M) = 0.0
      DO 83 J = 1,NC
      M = NP+NP+J
83    H1(I,M) = HC(I,J)
      DO 85 J = 1,NC
      M = NP+NP+NC+J
```

```
85     H1(I,M) = 0.0
       DO 812 J = 1,NC
       M = NP+NP+NC+NC+J
812    H1(I,M) = 0.0
80     CONTINUE
       DO 86 I = 1,NUP
       DO 87 J = 1,NP
       H2(I,J) = 0.0
       DO 87 K = 1,NP
87     H2(I,J) = H2(I,J)+ECCP(I,K)*PHITAU(K,J)
       DO 88 J = 1,NP
       M = NP+J
       H2(I,M) = 0.0
       DO 88 K = 1,NP
88     H2(I,M) = H2(I,M)+ECCP(I,K)*PM(K,J)
       DO 813 J = 1,NC
       M = NP+NP+J
813    H2(I,M) = 0.0
       DO 89 J = 1,NC
       M = NP+NP+NC+J
       H2(I,M) = 0.0
       DO 89 K = 1,NP
89     H2(I,M) = H2(I,M)+ECCP(I,K)*AM(K,J)
       DO 91 J = 1,NC
       M = NP+NP+NC+NC+J
91     H2(I,M) = HC(I,J)
86     CONTINUE
CCCCC  CALCULATE HA,HB AND PLB
       DO 92 I = 1,NUP
       DO 92 J = 1,NF
92     HA(I,J) = H1(I,J)-H2(I,J)
       DO 715 I = 1,NUP
       DO 715 J = 1,NF
       D(I,J) = 0.0
       DO 715 K = 1,NF
715    D(I,J) = D(I,J) + H1(I,K)*F(K,J)
       DO 93 I = 1,NUP
       DO 93 J = 1,NF
93     HB(I,J) = D(I,J)-H2(I,J)
       DO 716 I = 1,NUP
       DO 716 J = 1,NWP
       D(I,J) = 0.0
       DO 716 K = 1,NF
716    D(I,J) = D(I,J) + H1(I,K)*G(K,J)
       DO 94 I = 1,NUP
       DO 94 J = 1,NWP
94     PLB(I,J) = D(I,J)-PL(I,J)
CCCCC  CALCULATE PXSS AND WRITE
       IT = 2
       IMAX = 30
       CALL MMMT(G,W,GWG,NF,NWP,NFM,NWPM,NWPM,NFM,D,NFM)
       CALL MODCAL(F,GWG,PXSS,AM,PM,NF,NFM,IMAX,IT)
       WRITE(6,95)
95     FORMAT('0STEADY-STATE COVARIANCE OF STATES')
```

```
      DO 96 I = 1,NF
96    WRITE(6,915)(PXSS(I,J),J=1,NF)
CCCCC CALCULATE PEAXSS
      CALL MMMT(HA,PXSS,HPH,NUP,NF,NUPM,NFM,NFM,NUPM,D,NFM)
      CALL MMMT(PLB,W,PLWPL,NUP,NWP,NUPM,NWPM,NWPM,NUPM,
     1 D,NFM)
      DO 97 I = 1,NUP
      DO 97 J = 1,NUP
97    PEAXSS(I,J) = HPH(I,J)+PLWPL(I,J)
      WRITE(6,98)
98    FORMAT('OSTEADY-STATE COVARIANCE OF E1')
      DO 99 I = 1,NUP
99    WRITE(6,915)(PEAXSS(I,J),J=1,NUP)
CCCCC CALCULATE AND WRITE PEBXSS
      CALL MMMT(HB,PXSS,HPH,NUP,NF,NUPM,NFM,NFM,NUPM,D,NFM)
      CALL MMMT(PLB,W,PLWPL,NUP,NWP,NUPM,NWPM,NWPM,NUPM,
     1 D,NFM)
      DO 100 I = 1,NUP
      DO 100 J = 1,NUP
100   PEBXSS(I,J) = HPH(I,J)+PLWPL(I,J)
      WRITE(6,101)
101   FORMAT('OSTEADY-STATE COVARIANCE OF E2')
      DO 102 I = 1,NUP
102   WRITE(6,915)(PEBXSS(I,J),J=1,NUP)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCC CALCULATE X(TIME) TO A UNIT-STEP INPUT FROM ZERO
CCCCC INITIAL CONDITIONS CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      WRITE(6,274) NTIME,NWRITE
274   FORMAT('OUNIT-STEP TIME RESPONSE FOR TAU POINT'/
     1 ' NO. OF T-INCREMENTED POINTS IN THE UNIT-STEP TIME
     2 RESPONSE = ',I5/' TIME RESPONSE TO BE WRITTEN
     3 EVERY ',I5,'*T SECONDS')
      TIME = 0.0
      DO 200 I = 1,NUP
200   YC2(I) = 0.0
      WRITE(6,201) TIME,(YC2(I),I=1,NUP)
201   FORMAT('OTIME=',G13.6,'YC2(TIME-T+TAU+DELAY) = ',
     2 G13.6)
      DO 202 I = 1,NUP
202   E1(I) = 0.0
      WRITE(6,203)(E1(I),I=1,NUP)
203   FORMAT(10X,'E1 = ',G13.6)
      WRITE(6,204)
204   FORMAT('0')
      DO 205 I = 1,NF
205   X(I) = 0.0
      WRITE(6,206) TIME,(X(I),I=1,NF)
206   FORMAT('OTIME=',G13.6,'X=',7G13.6,/,25X,7G13.6)
CCCCC CALCULATE YC1(TIME+DELAY)
      DO 207 I = 1,NUP
      Q = 0.0
```

```
      DO 208 J = 1,NF
208   Q = Q + H1(I,J)*X(J)
207   YC1(I) = Q
      WRITE(6,209)(YC1(I),I=1,NUP)
209   FORMAT(10X,'YC1(TIME+DELAY) = ',G13.6)
CCCCC CALCULATE E2
      DO 210 I = 1,NUP
210   E2(I) = YC1(I)-YC2(I)
      WRITE(6,211)(E2(I),I=1,NUP)
211   FORMAT(10X,'E2 = ',G13.6)
CCCCC CALCULATE YC2(TIME+TAU+DELAY)
      DO 217 I = 1,NUP
217   YC2(I) = 0.0
      DO 213 I = 1,NUP
      Q = 0.0
      DO 214 J = 1,NF
214   Q = Q + H2(I,J)*X(J)
213   YC2(I) = YC2(I) + Q
      DO 215 I = 1,NUP
      Q = 0.0
      DO 216 J = 1,NWP
216   Q = Q+PL(I,J)
215   YC2(I) = YC2(I) + Q
      WRITE(6,218)(YC2(I),I=1,NUP)
218   FORMAT(10X,'YC2(TIME+TAU+DELAY) =',G13.6)
CCCCC CALCULATE E1
      DO 219 I = 1,NUP
219   E1(I) = YC1(I)-YC2(I)
      WRITE(6,203)(E1(I),I=1,NUP)
      IWRITE=0
      THETA = 0.0
      DO 220 JT = 1,NTIME
      DO 221 I = 1,NF
      Q = 0.0
      DO 222 J = 1,NF
222   Q = Q+F(I,J)*X(J)
221   XW(I) = Q
      DO 223 I = 1,NF
      Q = 0.0
      DO 224 J = 1,NWP
224   Q = Q+G(I,J)
223   X(I) = XW(I)+Q
      TIME = TIME+T
CCCCC CALCULATE PITCH ANGLE(TIME+T/2.0)
      THETA = THETA+X(2)*T
CCCCC CALCULATE YC1(TIME)
      DO 225 I = 1,NUP
      Q = 0.0
      DO 226 J = 1,NF
226   Q = Q+H1(I,J)*X(J)
225   YC1(I) = Q
CCCCC CALCULATE E2
      DO 227 I = 1,NUP
227   E2(I) = YC1(I)-YC2(I)
```

```
CCCCC CALCULATE YC2(TIME+TAU+DELAY)
      DO 228 I = 1,NUP
228   YC2(I) = 0.0
      DO 229 I = 1,NUP
      Q = 0.0
      DO 230 J = 1,NF
230   Q = Q+H2(I,J)*X(J)
229   YC2(I) = YC2(I)+Q
      DO 231 I = 1,NUP
      Q = 0.0
      DO 232 J = 1,NWP
232   Q = Q+PL(I,J)
231   YC2(I) = YC2(I) + Q
CCCCC CALCULATE E1
      DO 245 I = 1,NUP
245   E1(I) = YC1(I)-YC2(I)
      IWRITE = IWRITE+1
      IF(IWRITE.NE.NWRITE) GO TO 220
      IWRITE = 0
      WRITE(6,206) TIME,(X(I),I=1,NF)
      WRITE(6,209) (YC1(I),I=1,NUP)
      WRITE(6,211)(E2(I),I=1,NUP)
      WRITE(6,218)(YC2(I),I=1,NUP)
      WRITE(6,203)(E1(I),I=1,NUP)
      WRITE(6,124) THETA
124   FORMAT('     PITCH ANGLE(TIME+T/2.0) = ',G13.6)
220   CONTINUE
      STOP
      END
```

```
     A7 MODEL --- 4TH ORDER PLANT, 1ST ORDER CONTROLLER

NO. OF PLANT STATES =  4
NO. OF PLANT INPUTS =   1
NO. OF DISTURBANCE INPUTS =   1
NO. OF PLANT OUTPUTS =   3
NO. OF CONTROLLER STATES (EACH CONTROLLER) =   1

PLANT STATE MATRIX -- AP
 -.757200      1.00000      .0           -.109000
 -3.70120      -.551100     .0           -6.07340
  .0            .0          -2000.00      .0
  .0            .0           .0          -20.0000

PLANT CONTROL INPUT MATRIX -- B1P
  .0
  .0
  .0
  20.0000

PLANT DISTURBANCE INPUT MATRIX -- B2P
  .0
  .0
  2000.00
  .0

PLANT OUTPUT MATRIX -- CP
  .0           1.00000      .0            .0
  7.07074      -.119810     .0           -.186760
  .0            .0          1.00000       .0

CONTROLLER STATE MATRIX -- FC
  .951220

CONTROLLER CONTROL INPUT MATRIX -- GC
  .0           .475907E-01 -.142772E-01

CONTROLLER OUTPUT MATRIX (STATES) -- HC
  .174533E-01

CONTROLLER OUTPUT MATRIX (INPUTS) -- EC
  .250000      .425690E-03 -.127707E-03
```

```
T =       0.0125
NT =       50
NDELAY=   30
NTAU =   10
T = SAMPLE RATE.
NT = NB. OF EVENLY-SPACED SUBINTERVALS INTO WHICH T IS DIVIDED.
DELTA = T/NT = INCREMENT USED IN THE NUMERICAL INTEGRATIONS.


DISTURBANCE COVARIANCE MATRIX -- W
  1.00000
   1 1600  800

                    TAU=(NTAU*T)/NT=    0.00250

                    DELAY=(XNDELA*T)/NT=    0.00750

PHIT
 .990389        .123972E-01   .0             -.163455E-02
-.458838E-01    .992885       .0             -.669037E-01
 .0             .0            .139001E-10     .0
 .0             .0            .0              .779015

PHITAU
 .998093        .249580E-02   .0             -.284189E-03
-.923735E-02    .998626       .0             -.147986E-01
 .0             .0            .673842E-02     .0
 .0             .0            .0              .951255

PHIT(T-DELAY)
 .996166        .498339E-02   .0             -.590915E-03
-.184443E-01    .997231       .0             -.288528E-01
 .0             .0            .454072E-04     .0
 .0             .0            .0              .904885

PSIT(T)
-.193525E-03
-.872368E-02
 .0
 .221245
```

```
PS1T(T-DELAY)
-.287953E-04
-.146747E-02
 .0
 .951662E-01

PS1T(TAU)
-.700829E-05
-.373146E-03
 .0
 .437715E-01

PS1T(DELAY)
-.664555E-04
-.324668E-02
 .0
 .139297

PS2T(TAU)
 .0
 .0
 1.01388
 .0

PS2T(DELAY)
 .0
 .0
 1.02075
 .0

PS2(T-DELAY)
 .0
 .0
 1.02071
 .0
```

```
G(T,TAU)
.0
.0
1.02075
.0
.0
.0
.0
.0
.0
.0
-.144753E-01
PL(T,TAU)
-.129479E-03

F(T,TAU)
 .990389      .123900E-01  .367736E-08 -.163455E-02 -.495716E-06 -.411648E-04  .210324E-07
 .130934E-07 -.502573E-06 -.287443E-05  .0
-.458882E-01  .992519      .187406E-06 -.669035E-01 -.218388E-04 -.181352E-02  .926586E-06
 .576831E-06 -.256122E-04 -.126634E-03  .0
 .0           .0           .139001E-10  .0           .0           .0           .0
 .0           .0           .0           .0
 .286445E-03  .237867E-01 -.121534E-04  .779007      .379396E-03  .315055E-01 -.160972E-04
-.100210E-04  .166096E-02  .219995E-02  .0
 1.00000      1.00000      1.00000      1.00000      .0           .0           .0
 .0           .0           .0           .0
 1.00000      1.00000      1.00000      1.00000      .0           .0           .0
 .0           .0           .0           .0
 1.00000      1.00000      1.00000      1.00000      .0           .0           .0
 .0           .0           .0           .0
 1.00000      1.00000      1.00000      1.00000      .0           .0           .0
 .0           .0           .0           .0
 .336501     -.570184E-02 -.142772E-01 -.888804E-02  .0           .0           .0
 .0           .951220      .0           .0
 .0           .0           .0           .0           .0           .0           .0
 .0           1.00000      .0           .0
 .335912     -.485416E-02 -.962058E-04 -.846603E-02 -.130545E-05 -.108406E-03  .553883E-07
 .344810E-07  .0          -.756973E-05  .951220
```

```
ITER=10

STEADY-STATE COVARIANCE OF STATES
 .291637E-01  .231938E-01  .426071E-17  .119365E-01  .657724E-01  .657724E-01  .657724E-01
 .657724E-01  .184638      .184052      .183585
 .231938E-01  .208859      .217135E-15 -.329128E-01  .179477      .179477      .179477
 .179477      .827483E-01  .804430E-01  .783813E-01
 .426071E-17  .217135E-15  83.3546     -.140813E-13  .115863E-08  .115863E-08  .115863E-08
 .115863E-08 -.165420E-10 -.229935E-21 -1.18206
 .119365E-01 -.329128E-01 -.140813E-13  .236479      .186073      .186073      .186073
 .186073     -.775758E-01 -.843710E-01 -.723429E-01
 .657724E-01  .179477      .115863E-08  .186073      83.8335      83.8335      83.8335
 83.8335     -.990935      .189809     -.933284
 .657724E-01  .179477      .115863E-08  .186073      83.8335      83.8335      83.8335
 83.8335     -.990935      .189809     -.933284
 .657724E-01  .179477      .115863E-08  .186073      83.8335      83.8335      83.8335
 83.8335     -.990935      .189809     -.933284
 .657724E-01  .179477      .115863E-08  .186073      83.8335      83.8335      83.8335
 83.8335     -.990935      .189809     -.933284
 .184638      .827483E-01 -.165420E-10 -.775758E-01 -.990935     -.990935     -.990935
-.990935      1.45795      1.44918      1.44580
 .184052      .804430E-01 -.229935E-21 -.843710E-01  .189809      .189809      .189809
 .189809      1.44918      1.45796      1.43764
 .183585      .783813E-01 -1.18206     -.723429E-01 -.933284     -.933284     -.933284
-.933284      1.44580      1.43764      1.45069

STEADY-STATE COVARIANCE OF E1
 .437769E-05

STEADY-STATE COVARIANCE OF E2
 .628582E-04
```

```
UNIT-STEP TIME RESPONSE FOR TAU POINT
NO. OF T-INCREMENTED POINTS IN THE UNIT-STEP TIME                    RESPONSE =  1600
TIME RESPONSE TO BE WRITTEN                          EVERY    800*T SECONDS

TIME=  .0       YC2(TIME-T+TAU+DELAY) =     .0
          E1 =    .0

TIME=  .0        X=   .0          .0          .0          .0          .0          .0
                                .0          .0          .0          .0          .0
          YC1(TIME+DELAY) =    .0
          E2 =    .0
          YC2(TIME+TAU+DELAY) = -.129479E-03
          E1 =    .129479E-03

TIME=   9.99986      X= -.146612      -.992159E-01  1.02075       .909194E-01  .865843      .865843
                                .865843      .865843     -1.31510     -1.31510     -1.31600
          YC1(TIME+DELAY) =   -.483307E-01
          E2 =  -.194088E-05
          YC2(TIME+TAU+DELAY) = -.483287E-01
          E1 =  -.194088E-05
    PITCH ANGLE(TIME+T/2.0) =   -1.09443

TIME=   19.9988      X= -.146612      -.992159E-01  1.02075       .909194E-01  .865843      .865843
                                .865843      .865843     -1.31510     -1.31510     -1.31600
          YC1(TIME+DELAY) =   -.483307E-01
          E2 = -.194088E-05
          YC2(TIME+TAU+DELAY) = -.483287E-01
          E1 =  -.194088E-05
    PITCH ANGLE(TIME+T/2.0) =   -2.08625
```

```
     A7 MODEL --- 4TH ORDER PLANT, 1ST ORDER CONTROLLER

NO. OF PLANT STATES =   4
NO. OF PLANT INPUTS =    1
NO. OF DISTURBANCE INPUTS =    1
NO. OF PLANT OUTPUTS =   3
NO. OF CONTROLLER STATES (EACH CONTROLLER) =    1

PLANT STATE MATRIX -- AP
 -.757200      1.00000      .0          -.109000
 -3.70120      -.551100     .0          -6.07340
  .0           .0        -2000.00       .0
  .0           .0           .0          -20.0000

PLANT CONTROL INPUT MATRIX -- B1P
  .0
  .0
  .0
  20.0000

PLANT DISTURBANCE INPUT MATRIX -- B2P
  .0
  .0
  2000.00
  .0

PLANT OUTPUT MATRIX -- CP
  .0           1.00000      .0          .0
  7.07074      -.119810     .0          -.186760
  .0           .0           1.00000     .0

CONTROLLER STATE MATRIX -- FC
  .951220

CONTROLLER CONTROL INPUT MATRIX -- GC
  .0           .475907E-01 -.142772E-01

CONTROLLER OUTPUT MATRIX (STATES) -- HC
  .174533E-01

CONTROLLER OUTPUT MATRIX (INPUTS) -- EC
  .250000      .425690E-03 -.127707E-03
```

```
T =       0.0125
NT =      50
NDELAY=   30
NTAU =    25
T = SAMPLE RATE.
NT = NR. OF EVENLY-SPACED SUBINTERVALS INTO WHICH T IS DIVIDED.
DELTA = T/NT = INCREMENT USED IN THE NUMERICAL INTEGRATIONS.

DISTURBANCE COVARIANCE MATRIX -- W
  1.00000
   1 1600  800

                      TAU=(NTAU*T)/NT=    0.00625

                      DELAY=(XNDELA*T)/NT=    0.00750

PHIT
  .990389        .123972E-01   .0             -.163455E-02
 -.458838E-01    .992885       .0             -.669037E-01
  .0             .0            .139001E-10     .0
  .0             .0            .0              .779015

PHITAU
  .995244        .622417E-02   .0             -.752379E-03
 -.230366E-01    .996498       .0             -.356124E-01
  .0             .0            .372737E-05     .0
  .0             .0            .0              .882601

PHIT(T-DELAY)
  .996166        .498339E-02   .0             -.590915E-03
 -.184443E-01    .997231       .0             -.288528E-01
  .0             .0            .454072E-04     .0
  .0             .0            .0              .904885

PSIT(T)
 -.193525E-03
 -.872368E-02
  .0
  .221245
```

```
PSIT(T-DELAY)
-.287953E-04
-.146747E-02
 .0
 .951662E-01

PSIT(TAU)
-.455759E-04
-.227365E-02
 .0
 .117510

PSIT(DELAY)
-.564555E-04
-.324668E-02
 .0
 .139297

PS2T(TAU)
 .0
 .0
 1.02075
 .0

PS2T(DELAY)
 .0
 .0
 1.02075
 .0

PS2(T-DELAY)
 .0
 .0
 1.02071
 .0
```

```
G(T,TAU)
 .0
 .0
 1.02075
 .0
 .0
 .0
 .0
 .0
 .0
 .0
-.145734E-01
PL(T,TAU)
-.130356E-03

F(T,TAU)
 .990389      .123900E-01  .367736E-08 -.163455E-02 -.495716E-06 -.411648E-04  .210324E-07
 .130934E-07 -.502573E-06 -.287443E-05  .0
-.458882E-01  .992519      .187406E-06 -.669035E-01 -.218388E-04 -.181352E-02  .926586E-06
 .576831E-06 -.256122E-04 -.126634E-03  .0
 .0           .0           .139001E-10  .0           .0           .0           .0
 .0           .0           .0           .0
 .286445E-03  .237867E-01 -.121534E-04  .779007      .379396E-03  .315055E-01 -.160972E-04
-.100210E-04  .166096E-02  .219995E-02  .0
 1.00000      1.00000      1.00000      1.00000      .0           .0           .0
 .0           .0           .0           .0
 1.00000      1.00000      1.00000      1.00000      .0           .0           .0
 .0           .0           .0           .0
 1.00000      1.00000      1.00000      1.00000      .0           .0           .0
 .0           .0           .0           .0
 1.00000      1.00000      1.00000      1.00000      .0           .0           .0
 .0           .0           .0           .0
 .336501     -.570184E-02 -.142772E-01 -.888804E-02  .0           .0           .0
 .0           .951220      .0           .0
 .0           .0           .0           .0           .0           .0           .0
 .0           1.00000      .0           .0
 .335032     -.358743E-02 -.532164E-07 -.789471E-02 -.315083E-05 -.261648E-03  .133685E-06
 .832231E-07  .0          -.182702E-04  .951220

IHO208I IBCOM - PROGRAM INTERRUPT (P) -  UNDERFLOW   OLD PSW IS  078D000DA208AB36 . REGISTER CONTAINED
```

ITER=10

STEADY-STATE COVARIANCE OF STATES

.291637E-01	.231938E-01	.426071E-17	.119365E-01	.657724E-01	.657724E-01	.657724E-01
.657724E-01	.184638	.184052	.184364			
.231938E-01	.208859	.217135E-15	-.329128E-01	.179477	.179477	.179477
.179477	.827483E-01	.804430E-01	.820522E-01			
.426071E-17	.217135E-15	83.3546	-.140813E-13	.115863E-08	.115863E-08	.115863E-08
.115863E-08	-.165420E-10	-.229935E-21	-1.19006			
.119365E-01	-.329128E-01	-.140813E-13	.236479	.186073	.186073	.186073
.186073	-.775758E-01	-.843710E-01	-.728508E-01			
.657724E-01	.179477	.115863E-08	.186073	83.8335	83.8335	83.8335
83.8335	-.990935	.189809	-.928886			
.657724E-01	.179477	.115863E-08	.186073	83.8335	83.8335	83.8335
33.8335	-.990935	.189809	-.928886			
.657724E-01	.179477	.115863E-08	.186073	83.8335	83.8335	83.8335
83.8335	-.990935	.189809	-.928886			
.657724E-01	.179477	.115863E-08	.186073	83.8335	83.8335	83.8335
83.8335	-.990935	.189809	-.928886			
.184638	.827483E-01	-.165420E-10	-.775758E-01	-.990935	-.990935	-.990935
-.990935	1.45795	1.44918	1.44853			
.184052	.804430E-01	-.229935E-21	-.843710E-01	.189809	.189809	.189809
.189809	1.44918	1.45796	1.44028			
.184364	.820522E-01	-1.19006	-.728508E-01	-.928886	-.928886	-.928886
-.928886	1.44853	1.44028	1.45634			

STEADY-STATE COVARIANCE OF E1
.228205E-04

STEADY-STATE COVARIANCE OF E2
.254055E-04

```
UNIT-STEP TIME RESPONSE FOR TAU POINT
NO. OF T-INCREMENTED POINTS IN THE UNIT-STEP TIME                RESPUNSE =  1600
TIME RESPONSE TO BE WRITTEN                          EVERY   800*T SECONDS

TIME=  .0          YC2(TIME-T+TAU+DELAY) =    .0
         E1 =    .0


TIME=  .0          X=   .0             .0             .0             .0             .0             .0
                                  .0             .0             .0             .0             .0
         YC1(TIME+DELAY) =    .0
         E2 =    .0
         YC2(TIME+TAU+DELAY) = -.130356E-03
         E1 =    .130356E-03

TIME=  9.99936     X= -.146612     -.992159E-01  1.02075      .909194E-01  .865843      .865843
                                .865843      .865843      -1.31510      -1.31510      -1.31733
         YC1(TIME+DELAY) =  -.483307E-01
         E2 =    .294447E-04
         YC2(TIME+TAU+DELAY) = -.483601E-01
         E1 =    .294447E-04
   PITCH ANGLE(TIME+T/2.0) =  -1.09443

TIME=  19.9988     X= -.146612     -.992159E-01  1.02075      .909194E-01  .865843      .865843
                                .865843      .865843      -1.31510      -1.31510      -1.31733
         YC1(TIME+DELAY) =  -.483307E-01
         E2 =    .294447E-04
         YC2(TIME+TAU+DELAY) = -.483601E-01
         E1 =    .294447E-04
   PITCH ANGLE(TIME+T/2.0) =  -2.08625
```

References

1. Darcy, V. J. and C. Slivinsky, "Analysis of Inherent Errors in Asynchronous Redundant Digital Flight Control System," Technical Report AFFDL-TR-76-16, Air Force Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, April 1976.

2. John M. Wozencraft and Irwin Mark Jacobs, "Principles of Communication Engineering," John Wiley and Sons, Inc., 1965.

CHRONOLOGICAL LIST OF PUBLICATIONS

1. Darcy, V.J. and C. R. Slivinsky Analysis of Inherent Errors in Asynchronous Redundant Digital Flight Control Systems Technical Report AFFDL-TR-76-16, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, April, 1976.

2. Darcy, V.J. and C. Slivinsky, "Inherent Errors in Asynchronous Redundant Digital Flight Controls," IEEE Transactions on Automatic Control, February, 1977.

3. Slivinsky, C. and W. Shoemaker, "Comparison Monitoring in Redundant Digital Flight Control Systems" NAECON, May, 1978. To be submitted to the IEEE Transactions on Automatic Control.

4. Slivinsky, C. and S. Patumtawapibal "Models for Closed Loop Operation of Asynchronous, Redundant Digital Flight Control Systems," To be submitted to the IEEE Transactions on Automatic Control.

PROFESSIONAL PERSONNEL

This research is directed by Dr. Charles Slivinsky, Professor of Electrical Engineering, University of Missouri - Columbia, Columbia, Mo. 65201.

Students supported by this grant are as follows:

Student	Semesters Supported	Degree/Data
Timothy Holmes	Winter 1976	BS(EE)/June, 1976
Byung Ju Min	Winter 1976	PhD(EE)/June, 1976
Wayne Shoemaker	Summer 1977 Fall 1977	MS(EE)/June, 1977
Sudhiporn Patumtawapibal	Fall 1976 Winter 1977 Summer 1977 Fall 1977	MS(EE)/June, 1977

COUPLING ACTIVITIES

This research originated under an AFOSR program, namely, the 1975 USAF-ASEE Summer Faculty Research Program (June 9 - Aug. 15). Professor Slivinsky was one of 22 participants and was assigned to AFFDL/FGL (DAIS). He was assisted by Major Vincent J. Darcy. Together they devised a set of differential equations to model a dual-redundant, closed-loop flight control system with a simple voting algorithm. A software package was written to allow parametric analyses of the effects of design parameters on both transient and steady-state inherent errors.

AFOSR sponsored the continuation of this research with a grant to University of Missouri-Columbia (AFOSR-76-2968, 1 Feb. 76 - 31 Jan. 77). Professor Slivinsky (at UMC) and Major Darcy (at AFFDL) verified the model and software and applied both to a version of the A-7D flight control system. They found that inherent errors were on the order of 3% of command inputs. While small, these errors are not negligible, as their characteristics must be known to specify the algorithms that isolate failed signals and determine the best output from among the unfailed, redundant signals. This work resulted in an AFFDL Technical Report (AFFDL-TR-76-16 dated April 1976) and a paper in the IEEE Transactions on Automatic Control (February 1977).

AFOSR renewed the grant for a second year (AFOSR-76-2968A, 1 Feb. 77 - 31 Jan. 78), to develop more powerful closed-loop models and software, and to study signal-selection algorithms. The last five months of this grant overlap with the 1977-78 Sabbatical Leave of Professor Slivinsky at AFFDL.

For the last two years, the Flight Dynamics Laboratory supported work related to the AFOSR-sponsored research. During the Summer of 1976, Professor Slivinsky spent six weeks as a Consultant (University of Dayton Contract F33615-76-C-3076) at AFFDL/FGL working under Dr. A. DeThomas. The major result was the report "Some Guidelines for Testing AFFDL DAIS Concepts," which was used in writing the DAIS Concepts Test Plan.

For the period Nov. 76 - July 77, Professor Slivinsky continued as a Consultant, working approximately one day per week. This work was mainly to follow the progress of the DAIS program, offer consultation, and special studies.

Since Sept. 77, Professor Slivinsky has been on Sabbatical Leave at AFFDL/FGL (DAIS). For the period Sept. 77 - Jan. 78, he spends one day per week on the AFOSR-sponsored research and the remainder on AFFDL related research and development. His specific tasks include writing the flight control software for the AFFDL DAIS Flight Engineering Facility and studying problems associated with using higher order languages in digital flight control. AFFDL provides half-time support during this period.